AVIATION
ISSN: 1648-7788 / eISSN: 1822-4180
2019 Volume 23 Issue 1: 1–6
https://doi.org/10.3846/aviation.2019.9746

NUMBER OF CONFLICTS AT THE ROUTE INTERSECTION – MINIMUM DISTANCE MODEL

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Received 07 April 2017; accepted 12 March 2019

Abstract. A conflict is an infringement of minimum separation between at least two aircraft. The model is based on these assumptions: aircraft fly on level straight line routes, only an infringement of the lateral separation is considered, deviations are excluded, aircraft at the same flight level fly the same average speed, and aircraft fly towards an intersection and may change direction after intersection. Hence, conflicts mainly occur owing to a loss of minimum separation between aircraft flying at the same flight level. Calculation of average number of potential conflicts is designated for long time interval; hence, aircraft velocity deviations are negligible. The mathematical model in this paper is intended to compare different alternatives of intersection configuration of air traffic services routes. The comparison is based on the results: an average number of potential conflicts per hour on intersection of routes, index of conflicts intensity, and intersection capacity.

Keywords: intersection, configuration, aircraft conflicts, horizontal separation, protected zone.

Introduction

Air transport significantly contributes to the world economy development. Therefore, it is very important to sustain its further resilience, ensure effective, ecological activities and mainly ensure safety.

In 2015, more than 3.5 billion passengers used scheduled air transport, the growth of scheduled air transport compared to the year 2014 is 6.4%, and the number of flights was 34 million.

Based on the air traffic development ICAO identified Performance Based Navigation (PBN) as the main priority worldwide. ICAO concentrated also on PBN implementation at international airports during Continuous Descent Operations and Continuous Climb Operations. The SESAR AMAN and DMAN concepts, Free Route Airspace and growing air traffic requires delineation of new routes (ICAO, 2016).

All the above facts lead to the need of conflict research on routes intersections before their implementation. The mathematical model "A number of conflicts on routes intersection – minimum distance model" in this paper allows to compare different alternatives of intersection configuration of air traffic services routes. The comparison is based on the results: average number of potential conflicts per hour on intersection of routes, index of conflicts intensity, and capacity of intersection. The results should help to choose the safest intersection configuration of routes (Bugaj, Novák, & Beno, 2005). According to our literature review we analysed the paper Framework for airspace planning and design based on conflict risk assessment. Part 3: Conflict risk assessment model for airspace operational and current day planning (Netjasov & Babić, 2013); as well as other research papers: Analysis of the contribution of flight plan route selection to delays and conflicts (Belle & Sherry, 2011), On the conflict frequency at air route intersections (Schmidt, 1977), which are similar to our research.

A conflict described in our paper is an event in which two or more aircraft experience a loss of minimum separation. A conflict occurs when the distance between aircraft in flight violates a prescribed minimum, usually considered as 5 nautical miles (9 km) of horizontal or 1000 feet of vertical separation in radar environment (ICAO, 2007). These distances define a volume of airspace surrounding the aircraft, which should not be infringed upon by any other aircraft.

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An aircraft operating at cruising levels fly horizontal trajectories and are separated vertically, it means that conflicts mainly occur due to a loss of minimum separation between aircraft flying at the same flight level.

1. Model

A conflict situation in a radar environment occurs when the radar separation between aircraft is less than the prescribed minimum. A conflict is an infringement of minimum separation between at least two aircraft. The model is based on these assumptions: aircraft fly on level straight line routes, only an infringement of the lateral separation is considered, deviations are excluded, aircraft at the same flight level fly the same average speed, and aircraft fly towards an intersection and may change direction after intersection. Hence, conflicts mainly occur due to a loss of minimum separation between aircraft flying at the same flight level. Calculation of average number of potential conflicts is designated for a long time interval; hence, aircraft velocity deviations are negligible. The assumptions can be summed up as follows:

1. Aircraft fly in flight altitudes and at flight levels.
2. Only infringement of lateral separation is considered.
3. Aircraft fly on a straight line, deviations are excluded.
4. Aircraft at the same flight level fly at the same average speed V.
5. Longitudinal separation is always assured by ATC.
6. Aircraft can change the direction after the intersection.

The mathematical model is intended to compare different alternatives of an intersection configuration.

1.1. Deriving the model

Conflict definition

Consider isosceles triangle AOD such that $|AO| = |OD| = l$, angle $\angle AOD = \delta$, $\delta \in (0;\pi)$. First, we consider a case when aircraft $\bar{a}$ starts to move at a given moment at the constant speed V from the point A towards the point O; at that very moment the aircraft $\bar{b}$ starts to move from the point O towards the point D at the same constant speed V. If the aircraft $\bar{a}$ flies from the point A distance u, then the same distance will be flown by the aircraft $\bar{b}$ towards the point D (Figure 1). The distance $d = d(u)$ between $\bar{a}$, $\bar{b}$ can be calculated by cosine law:

$$d^2 = (l-u)^2 + u^2 - 2(l-u)u\cos\delta =$$

$$2\cos\delta\left(u^2 - ul + \frac{1}{4}l^2 - \frac{1}{4}l^2\right) + 2\left(u^2 - ul + \frac{1}{4}l^2\right) + \frac{1}{2}l^2 =$$

$$2\cos\delta\left(u - \frac{l}{2}\right)^2 - \frac{1}{2}l^2\cos\delta + 2\left(u - \frac{l}{2}\right)^2 + \frac{1}{2}l^2 =$$

$$2(1+\cos\delta)\left(u - \frac{l}{2}\right)^2 + l^2\frac{1-\cos\delta}{2} =$$

$$2(1+\cos\delta)\left(u - \frac{l}{2}\right)^2 + l^2\sin^2\frac{\delta}{2}.$$

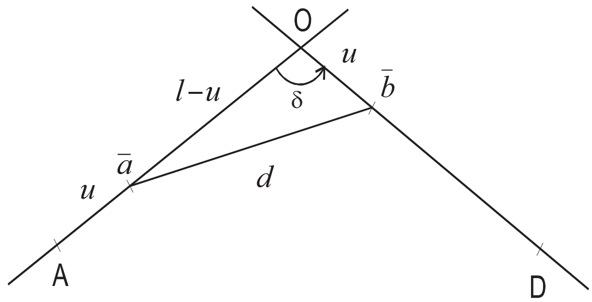

Figure 1. Conflict definition

Hence, $d(u)$ is minimal if $u = \frac{l}{2}$, whilst $d_{\min}\left(\frac{l}{2}\right) = l\sin\frac{\delta}{2}$. From this follows the supporting statement:

(1) If the minimum separation of the aircraft $\bar{a}$, $\bar{b}$ is at least $\kappa > 0$, then $l \cdot \sin\frac{\delta}{2} \geq \kappa$, and hence $l \geq \frac{\kappa}{\sin\frac{\delta}{2}} = \kappa \cdot \operatorname{cosec}\frac{\delta}{2}$.

The aircraft $\bar{a}$ and $\bar{b}$ fly at the same constant speed V at the same level and $\kappa > 0$ is the minimum aircraft separation. The route of the aircraft $\bar{a}$ is the refracted line AOB, the route of the aircraft δ is the refracted line COD, the routes will cross at the point O, the angle of crossing is α (Figure 2). The angles are as usual positive if they are anticlockwise. At the point O the aircraft $\bar{a}$ will turn towards the point B by the oriented angle α. At the point O the aircraft $\bar{b}$ will turn towards the point B by the oriented angle β.

Firstly, let us consider a situation when the aircraft $\bar{a}$ is at the intersection O before the aircraft $\bar{b}$; from it follows that when the aircraft $\bar{b}$ is at the point O, the aircraft $\bar{a}$ is at the point M behind the intersection O. Now, according to (1):

if $|OM| \leq \frac{\kappa}{\left|\sin\frac{\pi+\alpha-\delta}{2}\right|} = \frac{\kappa}{\left|\cos\frac{\alpha-\delta}{2}\right|}$, then both aircraft $\bar{a}$ and $\bar{b}$ were in conflict,

and if $|OM| > \frac{\kappa}{\left|\cos\frac{\alpha-\delta}{2}\right|}$, then there was no conflict (and apparently there will be no future conflict).

Similar approach is applied when the aircraft $\bar{b}$ is at the intersection O before the aircraft $\bar{a}$; in that case $\bar{a}$ is at the point L in front of the intersection O. Again, in accordance with (1) it is true that

if $|LO| \leq \frac{\kappa}{\left|\sin\frac{\beta+\delta-\pi}{2}\right|} = \frac{\kappa}{\left|\cos\frac{\beta+\delta}{2}\right|}$, then both aircraft $\bar{a}$, $\bar{b}$ were in conflict,

and if $|LO| > \frac{\kappa}{\left|\cos\frac{\beta+\delta}{2}\right|}$, then there was no conflict (and apparently there will be no future conflict).

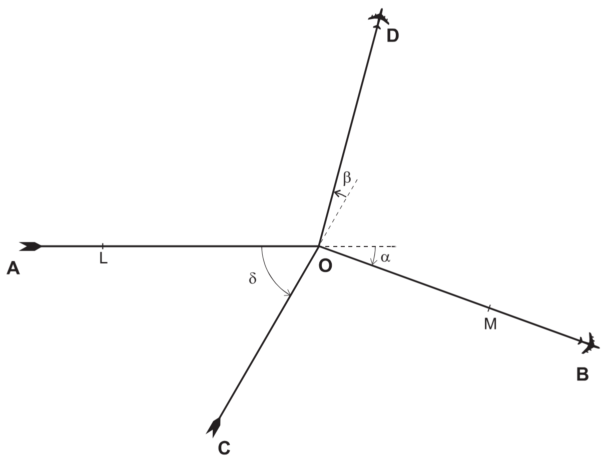

Figure 2. Intersection – angle

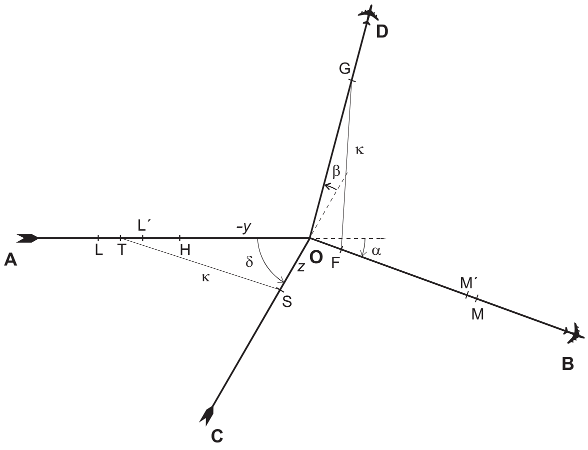

Figure 3. Intersection – duration conflict

If we locate the point $M \in OB$ such that $\left|OM\right| = \dfrac{\kappa}{\left|\cos\dfrac{\alpha-\delta}{2}\right|}$, and if the point $L \in AO$ such that $\left|LO\right| = \dfrac{\kappa}{\left|\cos\dfrac{\beta+\delta}{2}\right|}$, then the following statement (2) is true:

(2) Let the aircraft $\overline{b}$ in time t be at the point *O*. If in time t the aircraft $\overline{a}$ is anywhere on the refracted line *LOM*, then between the aircraft $\overline{a}$, $\overline{b}$ there is conflict just now or was conflict in the past or there is no risk of future conflict. If in time t the aircraft $\overline{a}$ is outside the refracted line *LOM*, then between aircraft $\overline{a}$, $\overline{b}$ there is no conflict just now, there was no conflict in the past and there is no risk of future conflict.

In order to devise the duration of a conflict we will concentrate on a situation, when in time $t = 0$ the aircraft $\overline{b}$ is at the point O and the aircraft $\overline{a}$ at the point H somewhere on the refracted line *LOM*. So

$$H = H(y)\text{, where } y \in \left\langle -\frac{\kappa}{\left|\cos\dfrac{\delta+\beta}{2}\right|}; \frac{\kappa}{\left|\cos\dfrac{\delta-\alpha}{2}\right|} \right\rangle.$$

Let the point L' on the abscissa LO be such that $\left|L'O\right| = \kappa$ and the point $M' \in OM$ be such that $\left|OM'\right| = \kappa$. The points L', O and M' divide the refracted line LOM into 4 segments (Figure 3) (Havel et al., 1990).

Consider a situation when the point H is on the abscissa $L'O$ so that $\left|HO\right| = -y \le \kappa$, i.e. the aircraft $\overline{a}$ and $\overline{b}$ are in conflict and the conflict was even before. Let us move both aircraft $\overline{a}$ from H and $\overline{b}$ from O in time back so that $\overline{b}$ is in *S* and $\overline{a}$ is in *T*, where $\left|TS\right| = \kappa$; apparently $\left|OS\right| = \left|TH\right| = z$, because the aircraft move at the same speed. κ is the minimum separation between aircraft; before that situation there was no conflict. If both aircraft start to move forward from the points *T* and *S*, the distance between them will be immediately less than κ until the aircraft $\overline{a}$ reaches the point F and the aircraft $\overline{b}$ reaches the point *G*. Hence the conflict duration is the time the aircraft $\overline{a}$ needs to cover the distance $\left|TO\right| + \left|OF\right|$.

The cosine law for the triangle TOS implies

$$\kappa^2 = (z-y)^2 + z^2 - 2(z-y)z\cos\delta.$$

It can be modified to the quadratic equation

$$2(1-\cos\delta)z^2 - 2y(1-\cos\delta)z + y^2 - \kappa^2 = 0,$$

whose roots are

$$z_{1,2} = \frac{y}{2} \pm \frac{1}{2}\left|\operatorname{cotg}\frac{\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2\frac{\delta}{2} - y^2}.$$

The root z_2 is not convenient because $z_2 < 0$. Thus, we have $z = z_1 = \dfrac{y}{2} + \dfrac{1}{2}\left|\operatorname{cotg}\dfrac{\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2\dfrac{\delta}{2} - y^2}$. Because the speeds of both aircraft $\overline{a}$, $\overline{b}$ are the same, we have $\left|TH\right| + \left|HO\right| + \left|OF\right| = \left|SO\right| + \left|OG\right|$, hence $\left|HO\right| + \left|OF\right| = \left|OG\right|$, i.e. $\left|OG\right| = -y + u$, where $u = \left|OF\right|$ (Havel & Husarčík, 1989).

Analogically for the triangle GOF:

$$\kappa^2 = (u-y)^2 + u^2 - 2(u-y)u\cos(\delta+\beta-\alpha).$$

If we denote $\psi = \beta + \delta - \alpha$, we get the quadratic equation

$$2(1-\cos\psi)u^2 - 2y(1-\cos\psi)u + y^2 - \kappa^2 = 0,$$

whose roots are

$$u_{1,2} = \frac{y}{2} \pm \frac{1}{2}\left|\operatorname{cotg}\frac{\psi}{2}\right| \cdot \sqrt{\kappa^2 \sec^2\frac{\psi}{2} - y^2}.$$

The negative root u_2 is not convenient. Therefore

$$u = u_1 = \frac{y}{2} + \frac{1}{2}\left|\operatorname{cotg}\frac{\delta+\beta-\alpha}{2}\right| \cdot \sqrt{\kappa^2 \sec^2\frac{\delta+\beta-\alpha}{2} - y^2}.$$

From this for $H \in L'O$ we obtain the required length

$$\left|TO\right| + \left|OF\right| = z - y + u = \frac{1}{2}\left|\operatorname{cotg}\frac{\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2\frac{\delta}{2} - y^2} + \frac{1}{2}\left|\operatorname{cotg}\frac{\delta+\beta-\alpha}{2}\right| \cdot \sqrt{\kappa^2 \sec^2\frac{\delta+\beta-\alpha}{2} - y^2}.$$

Let us consider the situation when $H \in LL'$ (Figure 4), i.e. both aircraft are not yet in conflict, however, the conflict will occur later in accordance with **(2)**. Let us move $\overline{a}$ by distance z into the point T and the aircraft $\overline{b}$ by the same distance z into the point S so that $|TS| = \kappa$. If the aircraft $\overline{a}$ and $\overline{b}$ move from those points, the conflict will persist until the aircraft $\overline{a}$ reaches the point F and the aircraft $\overline{b}$ reaches the point G, $|FG| = \kappa$. The triangles TOS and GOF are identical. So, $|FO| = |TH| = z$ and hence

$0 < |TF| = -y - 2z$. Clearly, $z < -\dfrac{y}{2}$.

The cosine law for the triangle TOS implies

$$\kappa^2 = (-y-z)^2 + z^2 - 2(-y-z)\,z\cos\gamma,$$

where $\gamma = \pi - \beta - \delta$.

It can be modified to the quadratic equation

$$2(1+\cos\gamma)\,z^2 + 2y\,(1+\cos\gamma)\,z + y^2 - \kappa^2 = 0,$$

whose roots are

$$z_{1,2} = -\frac{y}{2} \pm \frac{1}{2}\left|\mathrm{tg}\frac{\gamma}{2}\right| \cdot \sqrt{\kappa^2 \operatorname{cosec}^2 \frac{\gamma}{2} - y^2}.$$

From this $z_1 > -\dfrac{y}{2}$, and therefore the root z_1 is not convenient. Therefore

$$z = z_2 = -\frac{y}{2} - \frac{1}{2}\left|\mathrm{cotg}\frac{\beta+\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\beta+\delta}{2} - y^2}.$$

For $H \in LL'$ we obtain

$$|TF| = -y - 2z = \left|\mathrm{cotg}\frac{\beta+\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\beta+\delta}{2} - y^2}.$$

If we locate the point $H = H(y)$ on the right of O, i.e. $y > 0$ (Figure 4), we can analogically devise the following equations for $|TF|$:

$$|TF| = \frac{1}{2}\left|\mathrm{cotg}\frac{\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta}{2} - y^2} + \frac{1}{2}\left|\mathrm{cotg}\frac{\delta+\beta-\alpha}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta+\beta-\alpha}{2} - y^2}$$

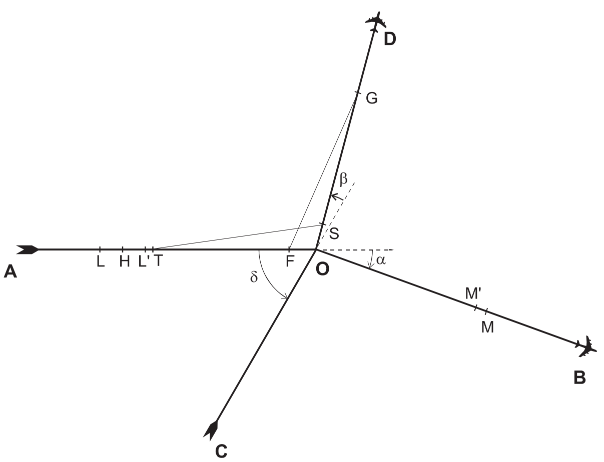

Figure 4. Intersection – duration conflict and capacity

for $H \in OM'$, i.e. for $y \in \langle 0; \kappa \rangle$ and

$$|TF| = \left|\mathrm{cotg}\frac{\delta-\alpha}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta-\alpha}{2} - y^2} \quad \text{for } H \in M'M,$$

i.e. for $y > \kappa$.

If ω denotes the length of the refracted line LOM, then $\omega = \dfrac{\kappa}{\left|\cos\frac{\delta-\alpha}{2}\right|} + \dfrac{\kappa}{\left|\cos\frac{\delta+\beta}{2}\right|}$. If μ denotes the mean value of the length of conflicts, then $\omega \cdot \mu = \int_{L}^{L'} |TF|\,dy + \int_{L'}^{O} |TF|\,dy + \int_{O}^{M'} |TF|\,dy + \int_{M'}^{M} |TF|\,dy$.

Substituting the computed values, we obtain

$$\omega \cdot \mu = \int_{\frac{-\kappa}{\left|\cos\frac{\delta+\beta}{2}\right|}}^{-\kappa} \left|\mathrm{cotg}\frac{\delta+\beta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta+\beta}{2} - y^2}\,dy + \int_{-\kappa}^{\kappa} \frac{1}{2}\left|\mathrm{cotg}\frac{\delta}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta}{2} - y^2}\,dy +$$

$$\int_{-\kappa}^{\kappa} \frac{1}{2}\left|\mathrm{cotg}\frac{\delta+\beta-\alpha}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta+\beta-\alpha}{2} - y^2}\,dy +$$

$$\int_{\kappa}^{\frac{\kappa}{\left|\cos\frac{\delta-\alpha}{2}\right|}} \left|\mathrm{cotg}\frac{\delta-\alpha}{2}\right| \cdot \sqrt{\kappa^2 \sec^2 \frac{\delta-\alpha}{2} - y^2}\,dy = \frac{1}{2}\left|\mathrm{cotg}\frac{\delta+\beta}{2}\right| \cdot$$

$$\left[\kappa^2 \sec^2 \frac{\delta+\beta}{2} \cdot \arcsin \frac{y}{\kappa\left|\sec\frac{\delta+\beta}{2}\right|} + y\sqrt{\kappa^2 \sec^2 \frac{\delta+\beta}{2} - y^2}\right]_{\frac{-\kappa}{\left|\cos\frac{\delta+\beta}{2}\right|}}^{-\kappa} +$$

$$\frac{1}{2}\left|\mathrm{cotg}\frac{\delta}{2}\right| \cdot \left[\kappa^2 \sec^2 \frac{\delta}{2} \cdot \arcsin \frac{y}{\kappa\left|\sec\frac{\delta}{2}\right|} + y\sqrt{\kappa^2 \sec^2 \frac{\delta}{2} - y^2}\right]_{0}^{\kappa} +$$

$$\frac{1}{2}\left|\mathrm{cotg}\frac{\delta+\beta\text{-}\alpha}{2}\right| \cdot$$

$$\left[\kappa^2 \sec^2 \frac{\delta+\beta-\alpha}{2} \cdot \arcsin \frac{y}{\kappa\left|\sec\frac{\delta+\beta-\alpha}{2}\right|} + y\sqrt{\kappa^2 \sec^2 \frac{\delta+\beta-\alpha}{2} - y^2}\right]_{0}^{\kappa} +$$

$$+\frac{1}{2}\left|\mathrm{cotg}\frac{\delta\text{-}\alpha}{2}\right| \cdot$$

$$\left[\kappa^2 \sec^2 \frac{\delta-\alpha}{2} \cdot \arcsin \frac{y}{\kappa\left|\sec\frac{\delta-\alpha}{2}\right|} + y\sqrt{\kappa^2 \sec^2 \frac{\delta-\alpha}{2} - y^2}\right]_{\kappa}^{\frac{\kappa}{\left|\cos\frac{\delta-\alpha}{2}\right|}} =$$

$$=\frac{1}{2}\left|\operatorname{cotg}\frac{\delta+\beta}{2}\right|\cdot$$

$$\left\{-\kappa^2\sec^2\frac{\delta+\beta}{2}\cdot\arcsin\left|\cos\frac{\delta+\beta}{2}\right|-\kappa^2\left|\operatorname{tg}\frac{\delta+\beta}{2}\right|+\kappa^2\sec^2\frac{\delta+\beta}{2}\cdot\frac{\pi}{2}\right\}+$$

$$\frac{1}{2}\left|\operatorname{cotg}\frac{\delta}{2}\right|\cdot\left\{\kappa^2\sec^2\frac{\delta}{2}\cdot\arcsin\left|\cos\frac{\delta}{2}\right|+\kappa^2\left|\operatorname{tg}\frac{\delta}{2}\right|\right\}+$$

$$+\frac{1}{2}\left|\operatorname{cotg}\frac{\delta+\beta-\alpha}{2}\right|\cdot$$

$$\left\{\kappa^2\sec^2\frac{\delta+\beta-\alpha}{2}\cdot\arcsin\left|\cos\frac{\delta+\beta-\alpha}{2}\right|-\kappa^2\left|\operatorname{tg}\frac{\delta+\beta-\alpha}{2}\right|\right\}+$$

$$+\frac{1}{2}\left|\operatorname{cotg}\frac{\delta-\alpha}{2}\right|\cdot$$

$$\left\{\kappa^2\sec^2\frac{\delta-\alpha}{2}\cdot\frac{\pi}{2}-\kappa^2\left|\operatorname{tg}\frac{\delta-\alpha}{2}\right|-\kappa^2\sec^2\frac{\delta-\alpha}{2}\cdot\arcsin\left|\cos\frac{\delta-\alpha}{2}\right|\right\}.$$

Apparently, $\left|\operatorname{cotg}x\right|\cdot\sec^2 x=\frac{1}{\left|\sin x\cos x\right|}=\frac{2}{\left|\sin(2x)\right|}$ and $\left|\operatorname{cotg}x\right|\cdot\left|\operatorname{tg}x\right|=1$.

Denoting $\Lambda(x)=\arcsin\left|\cos x\right|$, we obtain

$$\frac{\omega\mu}{\kappa^2}=\frac{\pi}{2}\cdot\left\{\frac{1}{\left|\sin(\delta+\beta)\right|}+\frac{1}{\left|\sin(\delta-\beta)\right|}\right\}+\frac{1}{\left|\sin\delta\right|}\cdot\Lambda\left(\frac{\delta}{2}\right)+$$

$$\frac{1}{\left|\sin(\delta+\beta-\alpha)\right|}\cdot\Lambda\left(\frac{\delta+\beta-\alpha}{2}\right)-\frac{1}{\left|\sin(\delta+\beta)\right|}\cdot\Lambda\left(\frac{\delta+\beta}{2}\right)-$$

$$\frac{1}{\left|\sin(\delta-\alpha)\right|}\cdot\Lambda\left(\frac{\delta-\alpha}{2}\right).$$

* * *

It is true that $\arcsin t=\frac{\pi}{2}-\arccos t$ for $t\in\langle-1,1\rangle$, hence $\Lambda(x)=\arcsin\left|\cos x\right|=\frac{\pi}{2}-\arccos\left|\cos x\right|$;

if $x\in\left\langle m\pi;m\pi+\frac{\pi}{2}\right\rangle$, then $\arccos\left|\cos x\right|=x-m\pi$ for every integer m;

if $x\in\left\langle m\pi-\frac{\pi}{2};m\pi\right\rangle$, then $\arccos\left|\cos x\right|=-x+m\pi$ for every integer m.

It means that $\arccos\left|\cos x\right|$ is a linear function, and hence, $\Lambda(x)$ is a linear function, too. Nevertheless, the reciprocal value of sinus plays the substantial role in the expression $\frac{\omega\mu}{\kappa^2}$.

2. Scientific result

Average number *E* of potential conflicts per hour

If the average speed is *V*, then the time of the flight on μ is $\frac{\mu}{V}$.

Let us denote

f_1 – average traffic flow on *AOB;*

f_2 – average traffic flow on *COD*.

Hence, the average occupancy time of aircraft from flow f_1 on μ is

$$T=\frac{f_1\mu}{V}.$$

During the time T we can expect $T\cdot f_2$ aircraft from f_2, so $E=T\cdot f_2$ is obviously the average number of potential conflicts per hour. So, we have

$$E=Tf_2=\frac{f_1 f_2\mu}{V}.$$

Index *I* of conflicts intensity

This index describes the intersection without the influence of the traffic flows

$$I=\frac{E}{f_1 f_2}=\frac{\mu}{V}.$$

Capacity *C* of intersection

The capacity for a given average number of potential conflicts per hour is

$$C=f_1 f_2=\frac{EV}{\mu}.$$

3. Discusion of the changes in route structures

Today, ICAO European Air Navigation Planning Group (ICAO EANPG) together with Eurocontrol prepare airspace design with the objective to create a dynamic airspace structure based on multi-option routeings and on areas of Free Route airspace operations. These changes in the dynamic airspace structure will have a negative impact on the scope of conflict, because its users plan a route between entry and exit point which creates number of unplanned crossings.

The index *I* of conflicts intensity evaluates, analyses and compares the route network before and after optimization. For deeper improvement of the route network the following must be done:

- general principles complemented by technical specifications for airspace design;
- an agreed route network and, where feasible, a free route airspace structure designed to meet al. users' requirements with details covering all the airspace change;
- route network and free route airspace utilisation rules and availability;
- indications on recommended Air traffic control (ATC) sectorization and sector families;
- guidelines for airspace management;
- an overview of the current and expected network situation, based on current and agreed plans.

Verification of the effectiveness method is not the primary goal of our research. Analysis of the reasons for deeper optimization is primary provided by Air Navigation Service Providers (ANSPs).

Conclusions

The mathematical model is intended to compare different alternatives of intersection configuration. The evaluation and comparison is based on the average number of potential conflicts per hour, index of conflict intensity, capacity of intersection. Calculation of average number of potential conflicts is designated for a long time-interval; hence aircraft velocity deviations are negligible. If the traffic flows on routes are constant, then the average number of potential conflicts per hour depends mainly on the mean value of length of conflicts. Decisive for index of conflict intensity of intersection is the mean value of length of conflicts. Capacity of intersection for a given number of potential conflicts per hour depends on the mean value of length of conflicts and on the average number of potential conflicts per hour.

Apart from a calculation of an average number of potential conflicts, a conflict probability estimation is also important. The growth of an air traffic intensity, a change of dynamics of relative aircraft movement and a reduction of separation should lead to an increased use of systems detecting and preventing conflicts and to a development of conflict probability estimation methods.

Nowadays the stochastic (probabilistic) methods of detecting and estimating conflict situations are considered the most promising. All known stochastic methods can be divided into two groups. One group is based on predicting the stochastic process of aircraft deviation from a planned trajectory and subsequently analysing the predicted relative position of the aircraft.

Another group of methods is based on the prediction of the aircraft position uncertainty area with subsequent analysis of their dangerous approach. The methods of this group do not give a sufficiently reliable result when the aircraft are approaching closer than 5 nautical miles. So these methods cannot estimate the risk of collision (Babak, Kharchenko, & Vasylyev, 2007).

Acknowledgements

This paper is published as one of the scientific outputs of the project: "Centre of Excellence for Air Transport ITMS 26220120065" and scientific outputs of the project: "New technologies and best practices in education in the Air Transport and Professional Pilots", KEGA 011ŽU-4/2018.

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Appendix

Legend

$\overline{a}$ – aircraft on the route AOB;
$\overline{b}$ – aircraft on the route COD;
δ – angle of arrival at intersection O;
β – oriented angle, the change of direction of flight of aircraft $\overline{b}$ in O;
α – oriented angle, the change of direction of flight of aircraft $\overline{a}$ in O;
κ – minimum aircraft separation;
V – speed of both aircraft (constant and the same);
μ – the mean value of length of conflicts.

The angles are positive if they are anticlockwise.