

## ANALYSIS OF CALCULATION RESULTS OF LIFT AND DRAG FORCES FOR SEVERAL WINGS USING NONLINEAR SECTION DATA

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**Abstract.** Calculation results for 11 different finite span wings are presented. Calculations were made by a combination of a numerical solution of lifting line theory with a technique developed to evaluate nonlinear section lift data. Aerodynamic coefficients for these wings are compared to the research results of other authors and to experimental data.

**Key words:** lift, drag, wing, non-linear aerodynamics.

## Introduction

Aerodynamic coefficients were determined for 11 different finite span wings. Comparisons to experimental data and to with results of calculations of other authors are presented. The calculations were made by a combination of the Vortex Step Method with a technique developed to evaluate nonlinear section lift data. Results include various flow conditions from  $Re=4000$  for a straight plate to  $Re=2,7 \times 10^6$  for a swept wing. Wings were chosen according to available experimental data.

## Review of methods using non-linear section lift data for calculation of lift and drag force along a wing span

Prandtl's Lifting Line Theory is the basis for most methods used to calculate change of lifting force of flat, low sweep angle and moderate and high aspect ratio wings. Tani developed the first successful technique in 1934 for handling nonlinear section lift-curve slopes in the formulation of Prandtl's Lifting Line Theory [17]. This method was made popular by the 1947 NACA (National Advisory Committee for Aeronautics) report of Sivells and Neely that provides a detailed description of the method for unswept wings with arbitrary planform and airfoil lift-curve slopes [15]. They apply this method for analysis of wings up to stall, i.e., until a wing angle of attack at any section on the wing has  $C_l$  equal to  $C_{lmax}$ .

Numerical solutions of Prandtl's Lifting Line Theory were also developed and are still in use. The most well known works using these methods are the research of McCormick and Anderson et al. [8, 1]. The method is also reflected in recent research of W. F. Phillips and D. O. Snyder [13].

The research of Mutterperl and Weisinger [19] made a base for the so-called Finite-Step Method or Vortex Step Method, which also was developed from Prandtl's Lifting Line Theory. Later Campbell and Blackwell simplified their method [10, 4, 3]. These methods presented the first attempts to couple sectional (two-dimensional) viscous results with inviscid wing (three-dimensional) theory. The most recent research reflecting their work has been made by Barnes J.P. [2]. He presents the Semi-Empirical Vortex Step Method, which includes empirical adjustments in lifting line position and shape.

Piszkin and Levinsky] developed a nonlinear lifting-line method based in part on the iterative method originally conceived by Tani [14, 17]. Their method differs from Prandtl's classical LLT in the implementation of the boundary condition. Tseng and Lan developed an entirely different approach to the use of nonlinear section data [18]. In their method, the reduction at any given wing section is determined by the difference

between the potential flow solution and the viscous  $C_l$  from the nonlinear section  $C_l$ - $\alpha$  curve.

In all methods using nonlinear section data the, main objective is that for the final solution of the three-dimensional flow, the  $\Gamma$  distribution across the span is consistent with the distribution of the effective  $\alpha$  for each section, and the  $C_l$  and  $C_m$  for each section is consistent with the effective  $\alpha$  for that section and the section  $C_l$ - $\alpha$  and  $C_m$ - $\alpha$  data. Mukherjee R., Gopalarathnam A., and Kim S W. [9] achieve that condition by finding the effective reduction in the camber distribution for each section along the span.

Another possibility to take into account is the nonlinear section data in wing calculation presented in the research of K. Jacob [6]. His method combines an inviscid 3d-lifting surface theory with a 2d-airfoil theory that includes boundary layer calculations and a displacement model for rear separation

In this research the vortex step method, which differs from the methods mentioned above by the nonlinear section data implementation technique, was used. Here an idea of E. Lasauskas was developed into a separate method for wing lift and drag force calculation using nonlinear section characteristics [7].

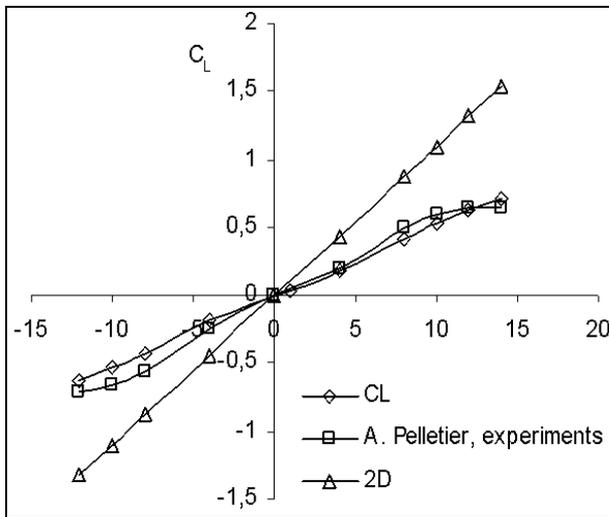
## Model of a finite wing

The wing model used is described in reference [11]. The method combines numerical solution of lifting line theory and a special approach to evaluate nonlinear section lift data. The iterative procedure allows wing lift at critical and post critical angles of attack to be predicted for the wings with moderate sweep and high aspect ratios.

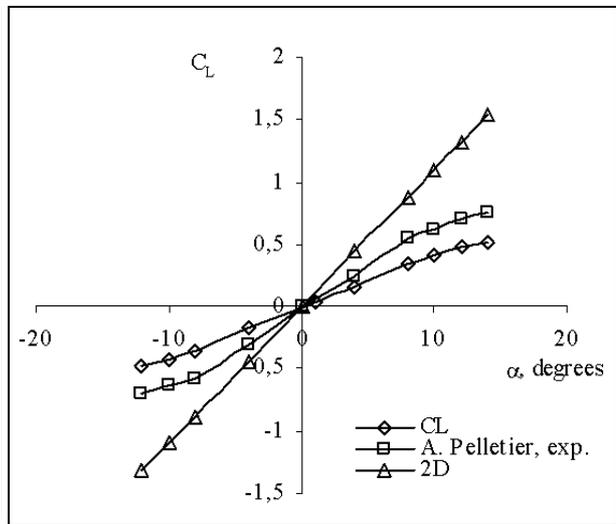
## Results

Figures 1 to 4 present calculation results of a rectangular flat plate of aspect ratio three at two different Reynolds numbers. Experimental results, presented in reference, were used as a section data [12]. These results were obtained in a low-speed, low-turbulence wind tunnel with a test section of 61 x 61 cm. Aluminium end plates were mounted in the test section. All wings tested were held at quarter-chord point, and a streamlined covering covered the sting. The gaps between the wing and the end plates were adjusted to approximately 0.8 mm. Uncertainty in the angle of attack was determined to be of 0.2 – 0.3 deg and 6% for  $C_l$  and  $C_D$ .

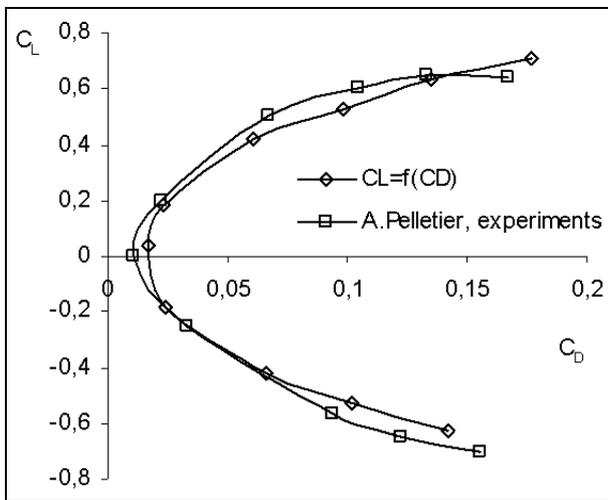
Figures 1 and 2 present lift and drag for a rectangular wing of a flat plate at Reynolds number 8000. Here it could be found that calculated curve presents the effect of finite wingspan compared to two-dimensional case. The results obtained are very close to the experimental results presented in ref. [12].



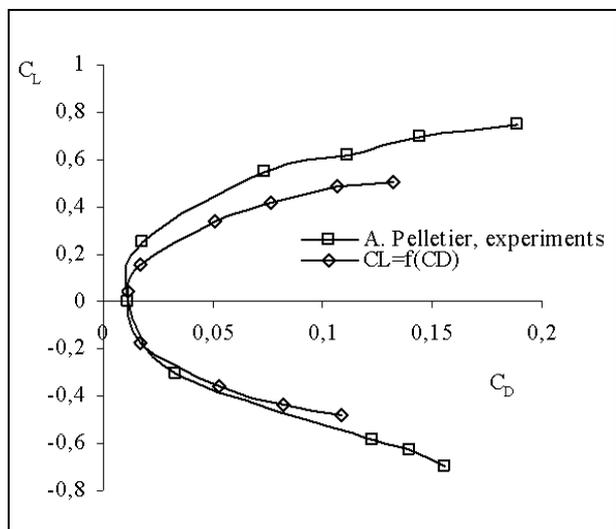
**Fig 1.** Lift coefficient of a flat plate rectangular wing of aspect ratio 3, at  $Re=80000$ . For calculation, section experimental results were used



**Fig 3.** Lift coefficient of a rectangular wing of aspect ratio 3 at  $Re=140000$ . For calculation, section experimental results were used



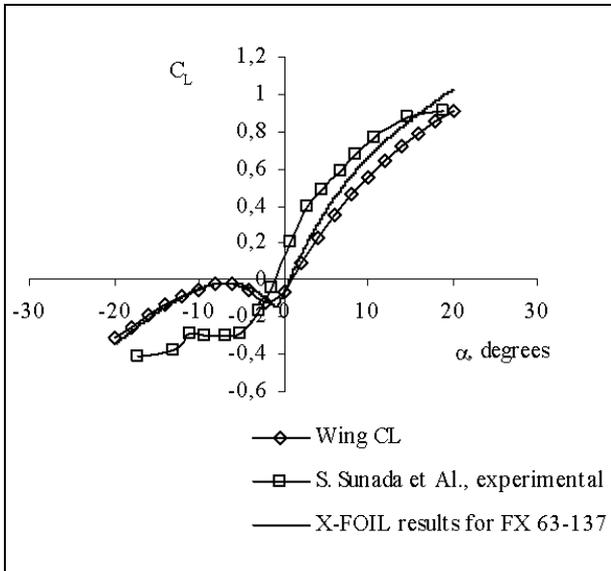
**Fig 2.** Lift-drag characteristics for a flat plate rectangular wing with aspect ratio 3 at  $Re=80000$ . For calculation, section experimental results were used



**Fig 4.** Lift-drag characteristics for a rectangular wing with aspect ratio 3 at  $Re=140000$ . For calculation, section experimental results were used

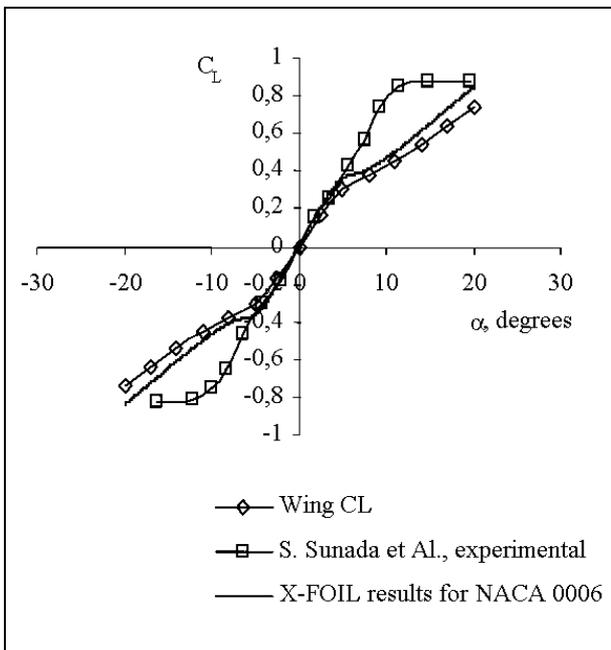
Figures 3 and 4 present the results for the same wing at Reynolds number 14000. It should be mentioned that the method itself does not account for the effect of the Reynolds number on the calculated wing characteristics. Here the change in the Reynolds number is evaluated in the 2D (sectional) characteristics, what means that a wing could be calculated for any Reynolds number for which 2D data is available. Compared to the results at  $Re=8000$ , bigger differences are noticed between calculated and experimental results. However, the lift curve slope over 5 degrees of angle of attack is still very close.

The following Figures 5-8 present another calculation attempt for a rectangular wing at ultra-low Reynolds numbers. Results are presented for several airfoils used for a wing of aspect ratio 7.25. Since experimental results were not available for such airfoils at  $Re=4000$ , section calculation results were made by means of X-FOIL [5]. As can be seen from the figures, calculation results are quite far from the experimental results, which are presented in ref. [16].

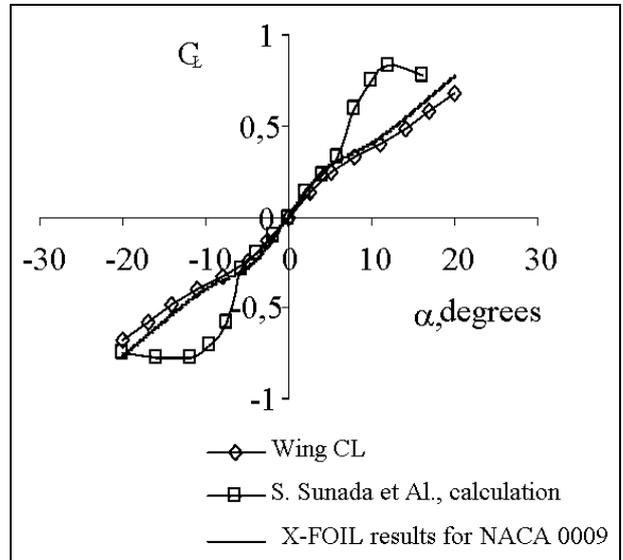


**Fig 5.** Lift coefficient of FX 63-137 rectangular wing AR=7.25 at Re=4000. X-FOIL calculation results were used as 2D section data

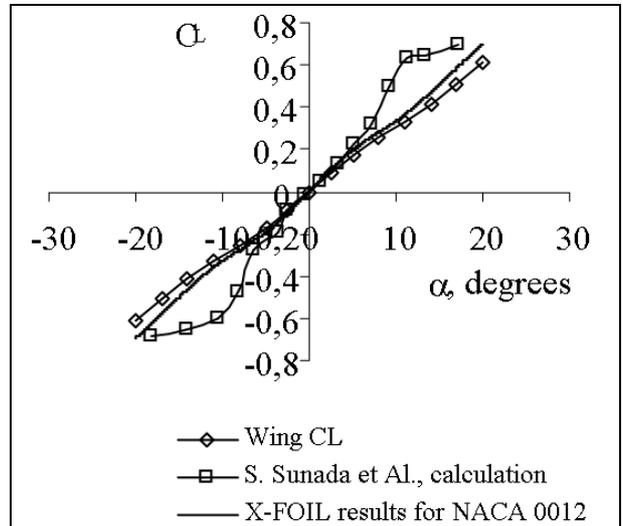
Such a situation was expected and is explained by the possibilities of X-FOIL. For such low Reynolds numbers X-FOIL cannot give suitable section characteristics.



**Fig 6.** Lift coefficient of NACA 0006 rectangular wing with AR=7.25 at Re=4000. X-FOIL calculation results were used as 2D section data



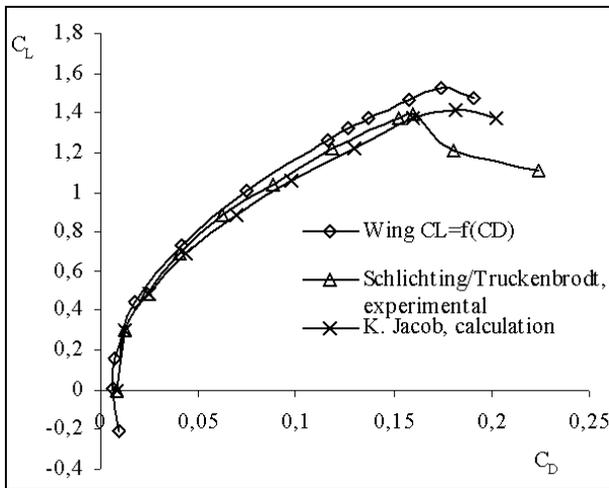
**Fig 7.** Lift coefficient of NACA 0009 rectangular wing with AR=7,25 at Re=4000. X-FOIL calculation results were used as 2D section data



**Fig 8.** Lift coefficient of NACA 0012 rectangular wing with AR=7.25 at Re=4000. X-FOIL calculation results were used as 2D section data

The results presented above make a proposal for more thorough research in the application of the method in the area of low Reynolds numbers. Due to lack of experimental data, it is not available at present time.

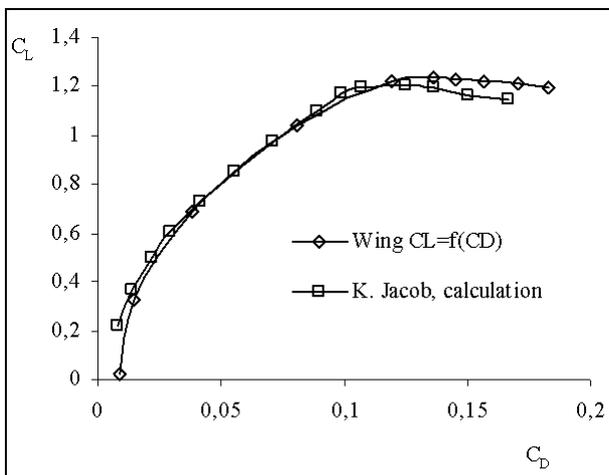
Figure 9 presents other calculation results for a rectangular wing with the use of X-FOIL sectional data. Here the results are compared with the results found by K. Jacob and with experimental results [6].



**Fig 9.** Lift-drag characteristics for a NACA 2412 rectangular wing with AR=5 at Re=2.7x10<sup>6</sup>. X-FOIL calculation results were used as 2D section data

As seen in Figure 9, there is good agreement between calculated and experimental results up to  $C_{Lmax}$ , but beyond it the experiments show a rapid decrease in lift. As this airfoil at calculated conditions develops a short bubble near the nose, this may be caused by the disappearance of the bubble [6]. Comparing the calculation results it is evident that the calculated lift curve slope is slightly higher than that obtained by K. Jacob, which results in higher  $C_{Lmax}$ . As previous research shows, the problem is in the different section data used [11]. K. Jacob used experimental section data whereas X-FOIL results were used in the present research. With the same source of section data, both methods present very close results. Such a situation can be noticed in figures 10 and 11, where the same experimental data were used by both methods.

Figure 11 presents the wing sweep evaluation results. Here calculation results for non-tapered NACA 4415 wing with 20 deg. of sweep back are presented. Close agreement with K. Jacob's results should be noticed.



**Fig 10.** Lift-drag characteristics for a NACA 4415 rectangular wing with AR=6.2 at Re=2.1x10<sup>6</sup>. Experimental results were used as 2D section data

For the clarity here the difference from Klaus Jacob's method should be explained. Jacob [6] uses the following, which is different from that one used in the present method, procedure to calculate the effective angle of attack [11].

Induced flow angles for each wing section are computed with 3-d lifting surface theory. Effective angle of attack is determined in that way:

$$\alpha_e = \alpha_g - \Delta\alpha$$

Here:

$$\Delta\alpha = F \cdot \Delta\alpha^{**} + (1 - F)\Delta\alpha^* ; F=0.33$$

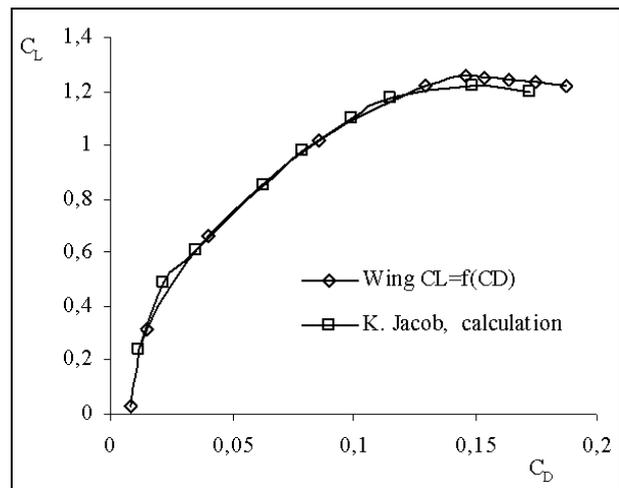
$$\Delta\alpha^* = \alpha^* - (\alpha^*)_{2-d}$$

$$\Delta\alpha^{**} = \alpha^{**} - (\alpha^{**})_{2-d}$$

Whereas induced angles of attack of the 3-d flow  $\alpha^*$  and  $\alpha^{**}$  are calculated from the system of linear equations based on the 3-d lifting surface theory, 2-d angles of attack are calculated in the following manner:

$$(\alpha^*)_{2-d} = (C_l - C_m) / 2\pi ;$$

$$(\alpha^{**})_{2-d} = (C_l + 8C_m) / 2\pi$$



**Fig 11.** Lift-drag characteristics for a NACA 4415 not tapered swept wing (20 deg.) with AR=6.2 at Re=2.1x10<sup>6</sup>. Experimental airfoil results were used as 2D section data

## Conclusions

1. Despite some results obtained for a flat plate at very low Reynolds numbers, the limits of the calculation method used depends on the availability of reliable section data.
2. Low Reynolds number experimental data is needed in order to evaluate the capabilities of the method at a very low Reynolds numbers.
3. For the moderate and high aspect ratios of rectangular wings the method provides results that

agree with the calculation results of K. Jacob and with experimental results.

4. Agreement with Jacob's results appears in the calculation of a sweptback wing, but comparison with experimental results is needed in order to evaluate the reliability of such results.

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