STRESS CONCENTRATION FACTOR OF FOAMED MATERIALS USED FOR CORES OF SANDWICH COMPOSITES

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Abstract. The numerical finite element method was used to identify the stress concentration factor of the structure of foamed polymer material under tensile loading by constant strain and depending on cell form, orientation, and geometrical parameters. It was determined that the value of the stress concentration factor depends upon both the form and the orientation of the cell with respect to the loading direction. It was also determined that the higher ratio of the thickness to the length of the edge of the cell is, the higher the stress concentration factor is. The low stress concentration factor is when the angle between edges is approximately 90° and the rounding radius of this angle is from 0.25*R* to 0.4*R* (*R* is maximal rounding radius), angle between cell edges, and the direction of loading is equal to 45°.

Keywords: stress concentration factor, foamed material, structure, cell geometry, finite element modelling, sandwich composites, core, matrix.

List of symbols

1. A, B, C, D = constants;
2. E = Young's modulus;
3. K_σ = stress concentration factor;
4. l_1 = length of cell edge;
5. l_2 = length of cell vertex;
6. r = rounding radius of angle α;
7. R = maximal rounding radius of angle α;
8. R^2 = determination factor;
9. S_{voids} = square of voids;
10. S_{model} = square of model;
11. t = thickness of cell edge;
12. α = angle between cell edges;
13. ε = strain;
14. γ_p = porosity;
15. μ = Poisson's ratio;
16. θ = angle of cell orientation;
17. σ_1 = principal stress.

Introduction

Foamed and honeycomb materials are currently widely used instead of monolithic materials, because they are cheaper, lighter, and exhibit good strength and deformability [5]. These materials, components, and their products are widely used in aviation, the automotive industry, packaging, and other applications.

An exceptional case of the use of foamed materials is sandwich composites with rigid matrixes and cores from foamed materials, frequently polymers. Such composites are ubiquitous in aircraft structures [4].

Foamed material is a heterogeneous system with a complex structure [7]. This system is a diphase composite with a solid matrix and gasiform filler [6]. The mechanical properties of heterogeneous systems depend not only on the nature of the material, but also on its structure. In the case of foams, mechanical behaviour is closely related to cell structure and geometry.

The aim of this investigation was to evaluate the influence of the structure of foamed material defined by cell form, orientation, and geometrical parameters on the value of the stress concentration factor.

1. Modelling

The structure of foamed materials was analysed by studying these materials with the help of a microscope and investigating photographs of the structure of foamed materials. It was determined that the typical structure consists of jointed edges that form polygonal elements, or so-called cells, oriented in various directions.

Finite element analysis was used to identify the influence of the structure of material and the direction of tension on stress concentration. Analysis was performed by the finite element code ALGOR [1]. A two-dimensional model was made to utilize symmetry and periodicity, assuming that there are no through-the-thickness stresses in the plane. The exact number of elements of each model depends on the type of model.

For the investigation of the influence of cell form and orientation on the stress concentration factor of such materials, the 2D models were created (Fig 1). Tetragonal (IA or IB), hexagonal (IIA or IIB), and octagonal (IIIA or IIIB) cells compose the models, and each model consists of nine cells. The length and thickness of the edges were $l_1 = 3.53$ mm and $t = 0.5$ mm. Cells IA, IIA and IIIA differ from IB, IIB and IIIB only in orientation: the first of them are rotated by angles of $\theta = 45°$, $\theta = 30°$ and $\theta = 45°$ respectively.

To obtain the influence of the geometrical parameters of the structure such as the angle α between edges, length of vertex l_2, rounding radius r of angle α, and thickness of edge t, on the stress concentration factor, three groups of models were created (Fig 2).

The first group of models was based on model IA and made with a variable angle α (Fig 2, a, b). In these models, the length of vertex l_2 was variable, but the length of edge l_1, thickness of edge t, and rounding radius r were constant (l_1 = 3.53 mm, t = 0.5 mm, r = 0 mm).

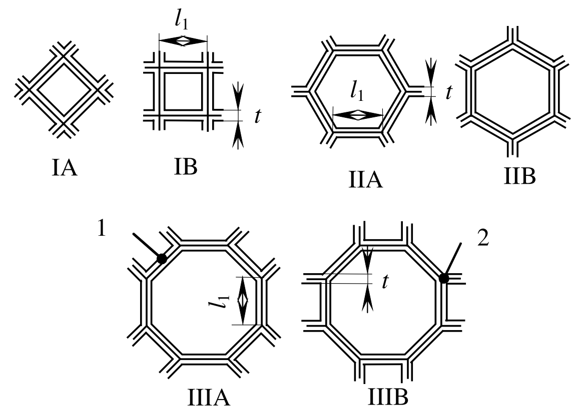


Fig 1. Cells of foamed materials; 1 – edge, 2 – vertex, l_1 – length of edge, t – thickness of edge

The second group was made on the basis of models IA and IB. The variable parameter of these models was rounding radius r of right angle α, and it was changed from 0 to R, i. e. to the full circle radius (Fig 2, c, d). The length of the edge was a constant l_1 = 3.53 mm as in the case of first group of models.

The third group (on the basis of IA) was created by changing the thickness of edges t and the relationship t/l_1 at the same time, but the other parameters were constant: r = 0 mm, α = 90°, and l_1 = 3.53 mm (Fig 2 e, f).

Every model investigated consisted of nine cells. Because the length of the model depended on the geometry of the cell, it was not constant for all models.

The porosity of each 2D model γ_p was calculated according to this equation

$$\gamma_p = \frac{S_{voids}}{S_{model}} \qquad (1)$$

in which S_{voids} and S_{model} are the squares of the voids and the model.

The boundary conditions were such that the upper surface was shear-free with a constant displacement constraint. The bottom surface had constraints on two directions at a point on the axis of symmetry of the model and constrains on one direction in other points. The right and left surfaces are assumed to be stress free. The strain of models was equal to 0.2. An enlarged description of the loading models was presented in previous investigations [10, 11].

Young's modulus of foam matrix material was E = 3.98 MPa and Poisson's ratio was μ = 0.46. Since this material was used for previous investigations, the choice of this material is done for comparison purposes.

The stress concentration factor K_σ was determined as the strength property of the model

$$K_\sigma = \frac{\sigma_1}{\sigma_{1N}}, \qquad (2)$$

where σ_1 is the maximal value of principal stress in the model and σ_{1N} is the nominal stress caused in pure matrix material under the same loading conditions.

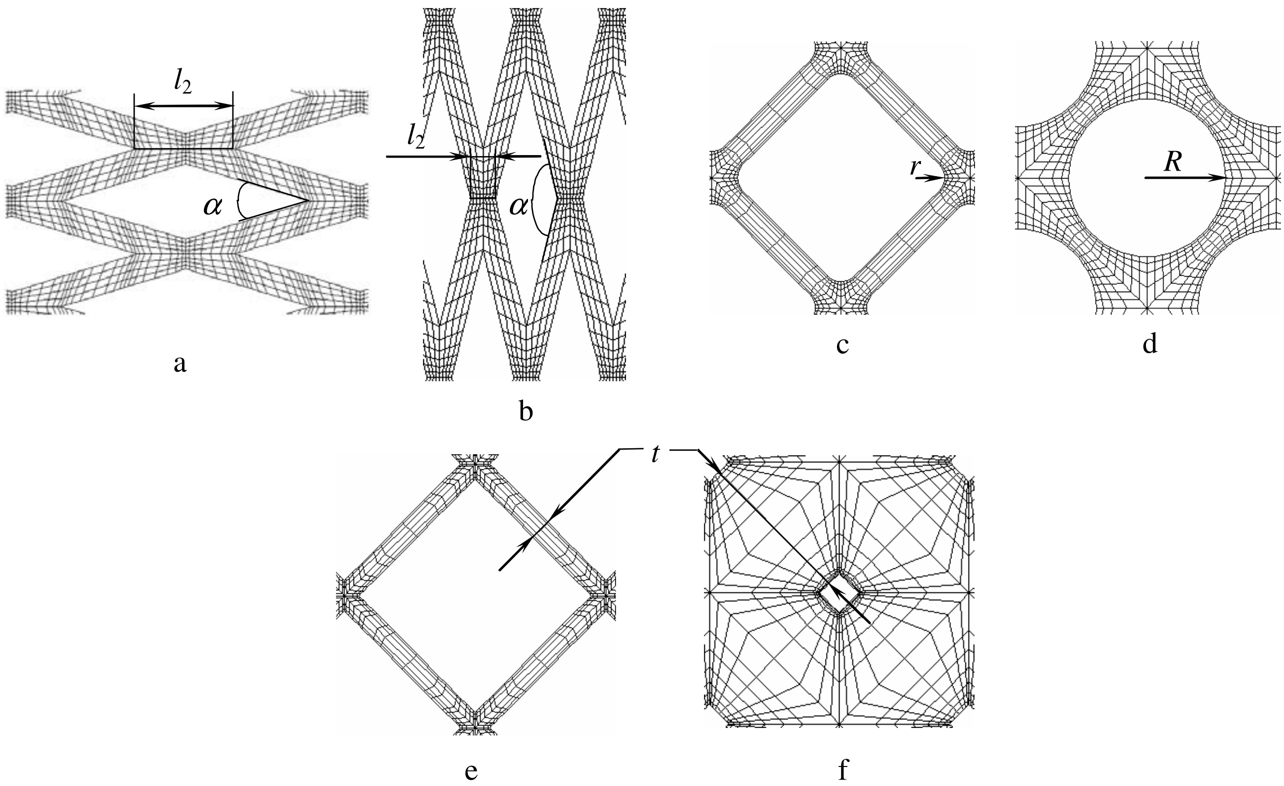


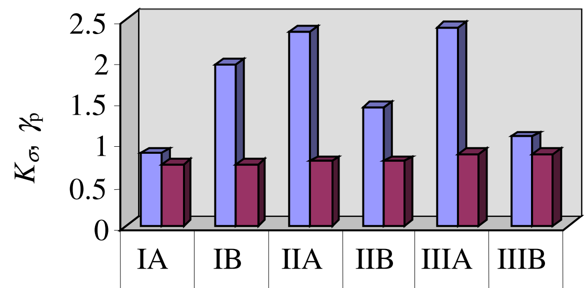
Fig 2. Numerical models: variable angle α between edges and variable length of vertex l_2 (a, b); variable rounding radius r of angle α (c, d); variable thickness of edge t (e, f)

2. Results and Discussion

2.1. Influence of cell form and orientation on stress concentration factor

The dependence of stress concentration factor K_σ and porosity value γ_p on cell form and orientation is presented in figure 3. It seems that the value of the stress concentration factor depends not only on the form of the cell but also substantially on the orientation. The lowest stress concentration factor is characteristic of model IA, but if it is transferred to IB, i. e. the orientation of IA is changed, the K_σ is significantly increased. As it was obtained before, the stress concentration factor depends upon the orientation of the edges of the matrix with respect to the direction of loading and on the changes of the stiffness of the adjacent zones of matrix in the case of the loading of the constant strain [10]. If the longitudinal axis of the thin edge is in line with the direction of tensile, the stress concentration factor is the highest.

If the angle between these edges and the tensile direction is equal to 45°, the stress concentration factor is the lowest. Therefore, the cell of the IA model consists of four edges oriented in direction of 45°. In the case of cell IB only the two edges that are parallel to the direction of tension experience loading. Due to this, the stress concentration factor of IB is more than two times higher that of IA.



	IA	IB	IIA	IIB	IIIA	IIIB
K_σ	0.88	1.95	2.35	1.43	2.39	1.08
γ_p	0.74	0.74	0.78	0.78	0.86	0.86

Fig 3. Stress concentration factor K_σ and porosity γ_p depending on model cell form and orientation the analogous reasoning, the behaviour of models IIA, IIB, and IIIA, IIIB can be explained

2.2. Influence of cell geometric parameters on stress concentration factor

The relationship of stress concentration factor K_σ and the ratio $l_2/(l_2(90°))$ with respect to the angle between edges α are presented in figure 4. The value of $l_2/(l_2(90°))$ defines the relation of the vertex lengths of the investigative model and that of basic model IA that exhibit angle α equal to 90°. The curve of $K_\sigma = f(\alpha)$ dependence could be described by the following equation

$$K_\sigma = A + \alpha\left(B + C\sqrt{\alpha}\right), \qquad (3)$$

where A, B, C are the constants: $A = 4.38$, $B = -0.53$, $C = 0.01$.

The dependence of $l_2/(l_2(90°)) = f(\alpha)$decreases in all investigated intervals of α, i. e. as the angle α increases, the length of vertex l_2 decreases. Due to this, the true area of the cross section of the cell also decreases. It is known from previous studies that in this case, the value of the deformation force decreases since the loading is defined by constant strain, and this leads to a decrease in the stress concentration factor [11]. But from figure 4 and equation (1), it is seen that $K_\sigma = f(\alpha)$ does not monotonically decrease in all cases of α investigated. Four zones are characteristic for this curve. In the zone I, where $0° < \alpha < 35°$, the stress concentration factor decreases due to a significant decrease in deformation force. In zone II ($35° < \alpha < 80°$), K_σ increases. That can be explained as follows. This abrupt decrease in deformation force is not characteristic of zone II and next zones, but the shape of the cell is such that the material becomes very similar to monolithic material with sharp cracks, and the zone near the crack tip decreases as the α increases. As is known from fracture mechanics, stresses increase in such cases [2]. Zone III can be defined as a plateau in which changes in stress are not significant. In this zone, α is approximate to 90° and the angle between the edges and the tensile direction is equal to 45°. This orientation of edges therefore leads to low stress concentration since porosity value is high. Since $\alpha > 120°$ (zone IV), the stress concentration factor significantly increases due to the significant decrease in vertex length and the orientation of the edges parallel to the loading direction.

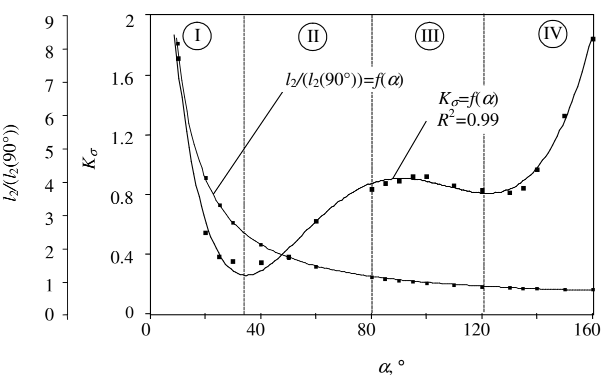


Fig 4. Relationship of stress concentration factor K_σ and the ratio $l_2/(l_2(90°))$ with respect to angle between edges α; R^2 is determination factor

In figure 5 the influence of rounding radius r of angle α=90° on the stress concentration factor K_σ is shown. It is readily seen that the dependence $K_\sigma = f(r/R)$ has an extreme mode with a minimum and can be described by this equation

$$K_\sigma = A + B\left(\frac{r}{R}\right)^2 + C\left(\frac{r}{R}\right)^3 + D\sqrt{\frac{r}{R}}, \qquad (4)$$

where A, B, C, D are constants: for models on the basis IA $A = 0.91$, $B = 1.37$, $C = -0.51$, $D = -0.65$; for models on the basis IB $A = 1.95$, $B = 0.43$, $C = 0.41$, $D = -0.91$.

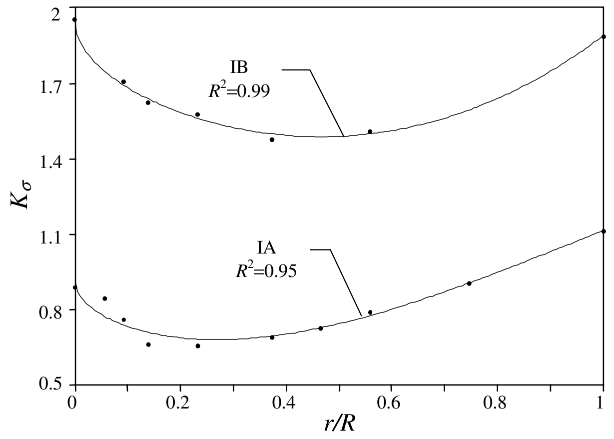


Fig 5. Relationship of stress concentration factor K_σ with respect to ratio r/R; R^2 is determination factor

As the rounding radius r increases to $0.25R$ in the case of IA and $0.4R$ in the case of IB, the stress concentration factor decreases. When r increases to $1R$, K_σ increases for both models IA and IB. It is known that stress concentration decreases as the rounding radius increases [8]. On the other hand, the abrupt changes of the stiffness of the adjacent of the matrix influence the high stress concentration factor [9]. In the case being investigated both the rounding radius and changes of the stiffness of the adjacent zones of the matrix influence the value of K_σ. As r increases to $0.25R \div 0.4R$, the changes of stiffness of the adjacent zones of the matrix are very insignificant, and K_σ therefore decreases. When r is higher, the changes of the stiffness of the adjacent zones of matrix becomes abrupt and due to this K_σ increases.

The investigation of models on the basis of IA with variable thickness of edge t showed that the higher ratio t/l_1 is, the higher the stress concentration factor is (Fig 6). The dependence $K_\sigma = f(t/l_1)$ can be described by the linear equation:

$$K_\sigma = A + B\frac{t}{l_1}, \qquad (5)$$

where A and B are the constants: $A = -0.11$ and $B = 6.02$.

When $t/l_1 \ll 1$, the model resembles the beam system that is formed from one-length and one-cross section slender beams connected at rigid joints. When stretching such a system, the beams experience not only tension but also bending or buckling. As is known from the mechanics of materials, when slender beams are bent, stress is low even though displacements are high [3]. Accordingly, the energy of deformation is used for bending in the event being investigated and due to this the stress concentration factor is as low as $t/l_1 \ll 1$. The edges lose the slenderness as the ratio t/l_1 increases. Therefore, they do not buckle because of the dominant tension. Due to this, the stress concentration factor increases. Besides, as the ratio t/l_1 increases, the porosity value of material decreases, so as obtained before, the appearance of small, infrequent pores results in the creation of a high stress concentration factor.

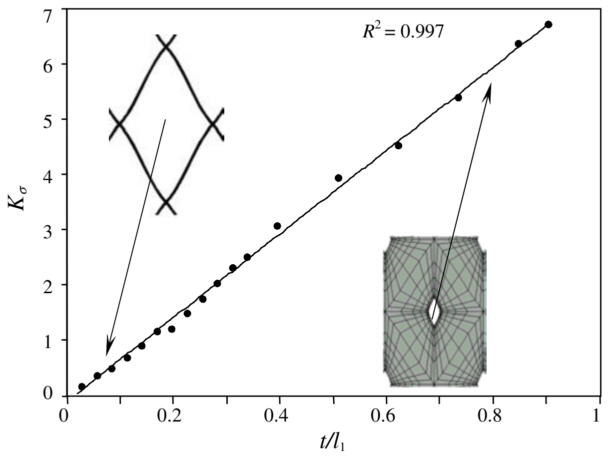


Fig 6. The influence of ratio t/l_1 on stress concentration factor K_σ; R^2 is determination factor

Conclusions

Stresses on foamed materials used for aircraft cores were analysed using the numerical finite element method.

It was determined that the stress concentration factor of the structure of foamed polymer material depends upon both the form and the orientation of the cell with respect to the loading direction in the case of the loading of the constant strain ($\varepsilon = 0.2$).

The higher ratio of the thickness to the length of the edge of the cell is, the higher the stress concentration factor is. The low stress concentration factor is when the angle between edges is approximately 90° and the rounding radius of this angle is from 0.25R to 0.4R (R is maximal rounding radius), angle between cell edges, and the direction of loading is equal to 45°.

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