DYNAMICS OF PLATE WITH POLYMERIC COATING

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Abstract. The paper considers the problem of suppressing vibration in a metal plate with polymeric coating. These types of plates are loaded with periodic variable force. The linear theory of plates and the method of complex numbers were used for calculations. The variation equation of oscillation was solved by the Ritz method. Optimum thickness of polymeric coating was determined for particular compositions of metal and plastics.

Keywords: composite material, polymeric coating, vibration attenuation, Hamilton's functional, Ritz method, flexural rigidity.

Introduction

Thin-walled structures of modern vehicles are subject to dynamic loading caused by the vibration of structural elements in operational conditions. Composite materials may be used to solve this problem. Compositions of metals and plastics are particularly effective for this purpose. Plastics are good thermal insulating materials, though lacking good mechanical properties. They can effectively suppress sound and vibration. Therefore, plastics may be used as a filling or coating.

In the present paper, the problem of vibration attenuation in a plate coated with polymeric material is considered. In the previous works of the authors, compositions of symmetrical structure were analysed [1–6].

Formulation of problem

A composition of a non-symmetrical structure consisting of a metal plate coated with polymeric material on one side is considered. This type of plate is loaded with periodic variable force. The problem is to calculate the equation of forced vibration, resonance amplitude, and optimum thickness of the polymeric layer.

Calculations are based on the linear theory of plates and the method of complex numbers. The variation equation of oscillation is solved by the Ritz method.

Equation of forced vibration of a plate

To determine the position of a neutral layer, plate rigidity should be calculated. The coordinate plane *x0y* was chosen to be at distance *z* from the support; see Fig 1. The thickness of the metal layer is t_1, and the thickness of the polymeric coating is t_2.

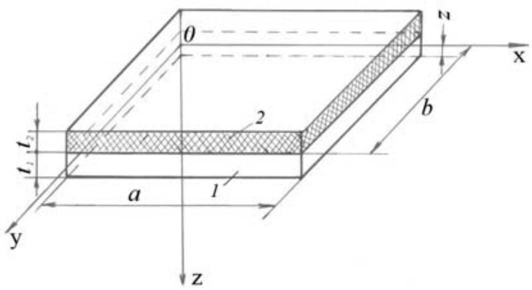

Fig 1. Two-layer plate: 1) supporting metal layer, 2) plastic coating

Flexural rigidity of the plate is expressed as follows:

$$D = D_1 + D_2 = \frac{E_1}{1-v_1^2}\int_{-z}^{(t_1-z)} z^2 dz + \frac{E_2}{1-v_2^2}\int_{(t_1-z)}^{t_2+t_1-z} z^2 dz =$$

$$= \frac{E_1}{3\left(1-v_1^2\right)}\left[(t_1-z)^3 + z^3\right] + \qquad (1)$$

$$+ \frac{E_2}{3\left(1-v_2^2\right)}\left[(t_2+t_1-z)^3 - (t_1-z)^3\right] \;,$$

where D_1, D_2, E_1, E_2, v_1, v_2 denote flexural rigidity, modulus of elasticity, and Poisson's ratio of metal and plastic layers, respectively.

The plate has the lowest rigidity in the neutral layer. To calculate the distance *z*, the minimum value of the function (1) should be found:

$$D = D_{\min}, \text{ when } \frac{dD}{dz} = 0 \,,$$

$$\frac{dD}{dz} = \frac{E_1}{1-v_1^2}\left[-(t_1-z)^2 + z^2\right] +$$

$$+ \frac{E_2}{1-v_2^2}\left[-(t_2+t_1-z)^2 + (t_1-z)^2\right] + 0 \,.$$

Allowing for an insignificant error, we can assume that $v_1 = v_2$.

Performing some mathematical operations, we get:

$$z = \frac{\frac{E_1}{E_2}t_1^2 + 2t_1t_2 + t_2^2}{2\left(\frac{E_1}{E_2}t_1 + t_2\right)} \;. \qquad (2)$$

The modulus of the elasticity of plastics makes only one tenth of that of metals. For example, it is $E_1 = 19.6 \cdot 10^{10}$ *Pa* for steel, while being $E_2 = 0.71 \cdot 10^{10}$ *Pa* for kapron. The ratio is $\frac{E_1}{E_2} = 27.6$. When both layers are of the same thickness ($t_1 = t_2$), we get z = 0.53 t_1, implying that the neutral layer is shifted upwards from the middle of the metal layer by 0.03 t_1. Since the thickness of the plate is small, this displacement will not be taken into account in further calculations. Thus, $z = 0.5\, t_1$ will be assumed.

The equation of forced oscillations is obtained as the required Hamilton functional

$$S = \int_{\tau_1}^{\tau_2} \left[T - (1+i\gamma)V + W\right] dt \;,$$

extremum condition:

$$\delta S = \delta \int_{\tau_1}^{\tau_2} \left[T_1 - (1+i\gamma)V + W\right] dt = 0 \,,$$

in the equation $T = T_0 \cdot e^{2i\omega t}$, $V = V_0 \cdot e^{2i\omega t}$ is the kinetic and potential energy of the plate, $W = W_0 \cdot e^{2i\omega t}$ is the work done by external forces, γ is the internal friction coefficient of the material, $\tau_2 - \tau_1$ is a short period of time, and *t* is time.

Since $e^{2i\omega t} \neq 0$, we finally get:

$$\delta\left[T_0 - (1+i\gamma)V_0 + W_0\right] = 0 \,. \qquad (3)$$

This variation equation, also taking into account inelastic forces, is solved by the Ritz method. A minimizing two-variable function is generated in this way:

$$w_0(x,y) = \sum a_{ij} w_{ij}(x,y), \qquad (4)$$

$$i = 1, 2, \dots n \,, \qquad j = 1, 2, \dots n$$

where $w_{ij}(x,y)$ denotes the parameters of the known basic function a_{ij}, whose values are obtained from the extremum condition as follows:

$$\frac{\partial}{\partial a_i}\left[T_0+(1+i\gamma)V_0+W_0\right]=0. \qquad (5)$$

When a harmonic external force acts on a rectangular plate of permanent thickness that is supported around the periphery, the following equations are obtained:

$$\left.\begin{aligned} T_0&=\frac{\mu\omega^2 h}{2}\int_0^a\int_0^b w_0^2\,dx\,dy,\\ W_0&=F_0\cdot w_0(x_F,y_F),\\ V_0&=\frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)^2 dx,dy.\end{aligned}\right\} \qquad (6)$$

For the two-layer plate we get:

$$T_0=T_{01}+T_{02}\,,\quad V_0=V_{01}+V_{02}\,. \qquad (7)$$

The data on the plate layers are as follows:
μ_1,μ_2 – mass of the unit area;
γ_1,γ_2 – the internal friction coefficient;
$h=t_1+t_2$ – the height (thickness) of the plate;
a,b – the length and width of the plate.

The data with the index "1" refer to the supporting metal layer, while those marked by "2" refer to the plastic layer.

Beam functions were chosen as basic functions for the rectangular plate supported around the periphery. For the main oscillation form (when $i = j = 1$) we get:

$$w_{ij}(x,y)=X_i(x)\cdot Y_j(y)=sin\frac{\pi x}{a}\cdot sin\frac{\pi y}{b}. \qquad (8)$$

By substituting the expression (4) into (6) and making the integration, we obtain:

$$\left.\begin{aligned} T_0&=\frac{\mu ab}{8}\omega_1^2 a_1^2,\\ V_0&=\frac{\pi^4 D_1\left(a^2+b^2\right)^2}{8a^3b^3}a_1^2+\frac{\pi^4 D_2\left(a^2+b^2\right)}{8a^3b^3}a_1^2,\\ W_0&=F_0 a_1\end{aligned}\right\} \qquad (9)$$

$$D=D_1+D_2=\frac{2E_1h_1^3}{3\left(1-\nu_1^2\right)}+\frac{E_2\left(h_2^3-h_1^3\right)}{3\left(1-\nu_1^2\right)},$$

$$h_1=\frac{t_1}{2},\quad h_2=\left(t_2+\frac{t_1}{2}\right).$$

The results (9) are substituted into equation (5):

$$\frac{\partial}{\partial a_1}=\left[\frac{\mu ab}{8}\omega_1^2a_1^2-\frac{\pi^4D_1\left(a^2+b^2\right)^2}{8a^3b^3}a_1^2-\frac{\pi^4D_2\left(a^2+b^2\right)}{8a^3b^3}a_1^2-i\gamma_1\frac{\pi^4D_1\left(a^2+b^2\right)^2}{8a^3b^3}a_1^2-i\gamma_2\frac{\pi^4D_2\left(a^2+b^2\right)^2}{8a^3b^3}a_1^2+F_0a_1\right]=0,$$

$$-a_1\left[1-\frac{\omega_1^2}{p_1^2}+i\left(\gamma_1\frac{D_1}{D}+\gamma_2\frac{D_2}{D}\right)\right]+\frac{4F_0a^3b^3}{\pi^4D\left(a^2+b^2\right)^2}=0$$

$$a_1=\frac{4F_0a^3b^3}{\pi^4D\left(a^2+b^2\right)}\cdot\frac{1}{1-\frac{\omega_1^2}{p_1^2}+i\left(\gamma_1\frac{D_1}{D}+\gamma_2\frac{D_2}{D}\right)}. \qquad (10)$$

In this formula

$p_1^2=\dfrac{4\pi^4D\left(a^2+b^2\right)^2}{\mu a^4b^4}$ is the basic natural frequency of the plate.

By transforming expression (10) into the index form $a_1=a_{01}e^{i\omega t}$, we obtain the amplitudes and the formulas for the initial stage (phase):

$$a_{01}=\frac{4F_0a^3b^3}{\pi^4D\left(a^2+b^2\right)}\cdot\frac{1}{\sqrt{\left(1-\frac{\omega_1^2}{p_1^2}\right)^2+\left(\gamma_1\frac{D_1}{D}+\gamma_2\frac{D_2}{D}\right)^2}}, \qquad (11)$$

$$\varphi=arc\,tg\frac{-\left(\gamma_1\frac{D_1}{D}+\gamma_2\frac{D_2}{D}\right)}{1-\frac{\omega_1^2}{p_1^2}}. \qquad (12)$$

By taking the real part of the complex function $w(x,y,t)=w_0(x,y)\cdot e^{i\omega t}$, we have the equation of forced oscillations for the two-layer plate as follows:

$$w=a_{01}\,sin\frac{\pi x}{a}\cdot sin\frac{\pi y}{b}\cdot cos(\omega t+\varphi). \qquad (13)$$

Optimum thickness of plate coating

To calculate the optimum thickness of the plate coating, the ratio of resonance amplitudes of the plate with or without coating is taken as:

$$\eta = \frac{a_{01}}{a_{01}^{*}} . \quad (14)$$

For the resonance case when $\omega_1 = p_1$, we obtain from formula (11) that

$$a_1 = \frac{4F_0 a^3 b^3}{\pi^4 (\gamma_1 D_1 + \gamma_2 D_2)(a^2 + b^2)^2} . \quad (15)$$

For the plate without coating $\gamma_2 = 0$, the amplitude is as follows:

$$a_{01}^{*} = \frac{4F_0 a^3 b^3}{\pi^4 \gamma_1 D_1 (a^2 + b^2)^2} . \quad (16)$$

We substitute formulas (15) and (16) into (14). Performing some mathematical operations, we have:

$$\eta = \frac{1}{1 + \frac{\gamma_2 D_2}{\gamma_1 D_1}} . \quad (17)$$

where, by expressing rigidity through the respective thickness of the layers in formula (17), we obtain the function $\eta = \eta\left(\frac{t_2}{t_1}\right)$ as follows:

$$\eta = \frac{1}{1 + \frac{\gamma_2 E_2}{2\gamma_1 E_1}\left[\left(\frac{2t_2}{t_1} + 1\right)^3 - 1\right]} . \quad (18)$$

Let us calculate the particular compositions:

1. Steel Cm3 with kapron coating.
2. Steel Cm3 with glass reinforced plastic coating.
3. Aluminium alloy D16A with kapron coating.
4. Aluminium alloy D16 with glass reinforced plastic coating.

The data on the materials [6]:
steel Cm3 – $E = 19.6 \cdot 10^{10}$ Pa, $\gamma = 0.003$;
alloy D16A – $E = 7.06 \cdot 10^{10}$ Pa, $\gamma = 0.0022$;
kapron $E = 0.71 \cdot 10^{10}$ Pa, $\gamma = 0.124$;
glass reinforced plastic $E = 1.5 \cdot 10^{10}$ Pa, $\gamma = 0.124$.

The results of the calculations are presented in the table and figure 2.

Table. Effect of thickness of coating on level of vibration

	Composition \ t_2 / t_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
η	Cm3 + kapron	1	0.646	0.433	0.310	0.216	0.160	0.121	0.094	0.074
	Cm3 + glass reinforced plastic	1	0.457	0.260	0.165	0.113	0.081	0.060	0.046	0.036
	D16A + kapron	1	0.333	0.172	0.105	0.070	0.049	0.036	0.028	0.021
	D16A + glass reinforced plastic	1	0.190	0.090	0.053	0.034	0.024	0.018	0.013	0.010

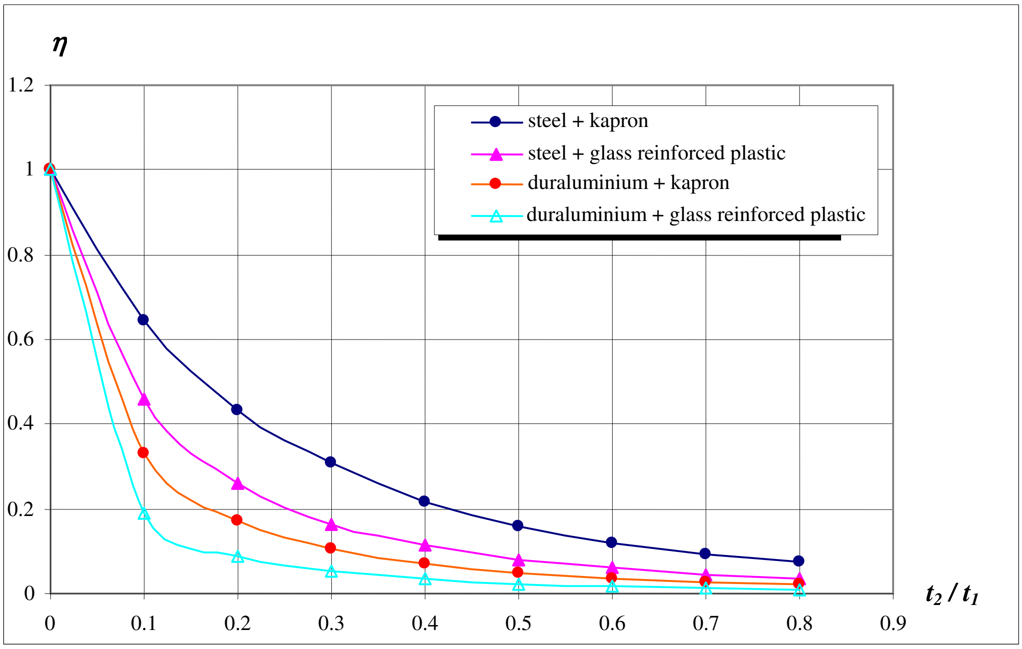

Fig 2. Effect of thickness of coating on level of vibration

Conclusions

1. As shown in figure 2, plastic coating considerably reduces oscillation amplitude.
2. The effectiveness of plastic coating depends on the product of γE: the larger the product, the higher the suppression of oscillations (for kapron $\gamma E = 0.088 \cdot 10^6$, while for glass reinforced plastic $\gamma E = 0.186 \cdot 10^6$).
3. The optimum ratio of the plate layers is in the range of $0.4 \div 0.7$.

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