AVIATION
ISSN 1648-7788 / eISSN 1822-4180
2016 Volume 20(1): 1–7
doi:10.3846/16487788.2016.1162197

MODELLING OF IONOSPHERIC TIME DELAYS BASED ON ADJUSTED SPHERICAL HARMONIC ANALYSIS

J R K Kumar DABBAKUTI[1], D. Venkata RATNAM[2], Surendra SUNDA[3]

[1,2]Department of ECE, KLEF, K L University, Vaddeswaram, Guntur Dt,522502, Andhra Pradesh, India
[3]Airports Authority of India, Ahmedabad, India, Andhra Pradesh, India
E-mails: [1]kirankumar.au@gmail.com; [2]dvratnam@kluniversity.in (corresponding author);
[3]ssunda@gmail.com

Received 08 July 2015; accepted 09 February 2016

D. Venkata RATNAM, *PhD.*
Education: MSc (Electronics) degree from Andhra University, Faculty of Physics, A.P, India; MTech (Radar and Microwave engineering) degree from Andhra University, A.P, India, Faculty of Engineering. 2001 to 2003, PhD studies, JNTUH University, Hyderabad, A.P, India, Faculty of Electronics and Communication Engineering 2007–2011.
Affiliation and functions: 2003 to 2011, research assistant, junior research fellow, senior research fellow, senior research assistant, Research and Training Unit for Navigational Electronics, Osmania University, Hyderabad, A.P, India;
Since 2011, Professor, Department of Electronics and Communication Engineering, KL University, Vaddeswaram, Guntur, A.P, India.
Research interests: navigation electronics, global positioning system, space science, radio wave propagation.
Publications: author of 12 international and 5 national research papers and 23 national and international conference papers.

J R K Kumar DABBAKUTI
Education: BTech (Electronics and Communication Engineering) from the Jawaharlal Technological University, Hyderabad, India in 2008; MTech (Electronics and Instrumentation) degree from Andhra University, A.P, India, Faculty of Engineering, 2010.
Affiliation and functions: 2010-2014, Assistant Professor, Department of Electronics and Communication Engineering, VIIT, Visakhapatnam, A.P, India.
Since 2014, Research Scholar, Department of Electronics and Communication Engineering, KL University, Vaddeswaram, Guntur, A.P, India.
Research interests: Global Positioning System (GPS), space science, communications.

Surendra SUNDA. *PhD.*
Education: B.Tech and MTech (Electronics and Communication Engineering), PhD.
Affiliation and functions: Manager at Airports Authority of India, Ahmedabad, India.
Research interests: Global Positioning System (GPS),GAGAN, GBAS, ionospheric modelling, aviation and aerospace.

Abstract. The ionosphere is the region of the upper atmosphere and the study of the upper atmosphere has a significant role in monitoring, modeling and forecasting for satellite based navigation services. As India lies in a low latitude region, a more careful approach has to be taken to characterize the ionosphere due to the irregularities and equatorial anomaly conditions. In order to study the ionospheric temporal variations, a regional ionospheric model based on the Adjusted Spherical Harmonic Analysis (ASHA) is implemented. The results indicate that the ASHA model is one of the contenders for estimating ionospheric delays well for GNSS augmentation systems.

Keywords: Global Position Systems (GPS), Total Electron content (TEC), Spherical Harmonic Analysis (SHA), Adjusted Spherical Harmonic Analysis (ASHA), Global Navigation Satellite Systems (GNSS).

1. Introduction

A stand alone Global Positioning System (GPS) does not meet the air navigation parameters, such as: continuity, integrity, availability, and accuracy. In order to use GPS for 'safety-of-life' applications, such as air navigation, augmentation (satellite based or ground based) is required to achieve the navigation parameters. A Satellite-Based Augmentation System (SBAS) provides a wide area navigation facility, whereas a Ground-Based Augmentation System (GBAS) is limited to a particular airport. The important aspect of the SBAS is to broadcast ionospheric corrections to the SBAS users through a geostationary satellite. Grid based ionospheric models are being implemented by several countries for their SBAS systems to estimate ionospheric delays. The present SBAS systems developed across the world are, US Wide Area Augmentation System (WAAS), European Geostationary Navigation Overlay System (EGNOS) in Europe, Japanese Multi-functional Satellite-Based Augmented Systems (MSAS) and India's GPS Aided GEO Augmented Navigation (GAGAN). The Russian SBAS – System for Differential Correction (SDCM) is under development. The Indian SBAS – GAGAN is being developed jointly by the Indian Space Research Organization (ISRO) and the Airport Authority of India (AAI). The Equatorial anomaly phenomenon covers a region from the magnetic equator to 30° geomagnetic latitudes in each hemisphere. The equatorial and low latitude regions are characterized by large temporal and spatial gradients in the ionospheric delays. Therefore, an appropriate ionospheric model is necessary to investigate the ionospheric conditions over India. Ionospheric delays are characterized based on the ionospheric grid point (IGP) and ionospheric pierce point (IPP) (Ratnam 2013). Grid-based models are generally used to estimate the ionospheric delays at the predefined ionospheric grid points and provide the values to the users (Sunda *et al.* 2015). Then the user receiver interpolates the broadcast vertical delay at the IGPs close to their ionospheric pierce point to estimate its ionospheric delay.

Spherical Harmonic Analysis (SHA) is a prominent model for modeling total electron content (TEC) data scattered over the sphere. The SHA is based on the Fourier expansion of orthogonal functions with the solution of Laplace's equations. While fitting the regional data to the globe, the SHA addresses the problem in regional modeling over the restricted area (Haines 1985). Therefore, a method is required for the modeling of ionospheric delay over a restricted area. A variety of nonharmonic methods have been developed for the assessment of ionospheric delay over a restricted area (Zolesi *et al.* 1993; Dvinskikh, Naidenova 1991). In the present study, the ionospheric characteristics of the regional spatial interpolation methods are considered. For modeling the regional geomagnetic field, various alternative approaches have been adopted (Molina, De Santis 1987). The individual elements of the local geomagnetic field are expected to represent the latitude and longitude by a simple polynomial modeling of the spherical harmonic expansion. This method requires the model coefficients to provide a realistic description of the regional field. Short-wavelength fields should be used and characterized with high degree polynomials. Unfortunately, they can produce duplicate values to fitting the data. Whereas, the data dissemination within an area must be identical (De Santis *et al.* 1990). Hence, a method for regional geomagnetic field modeling based on the rectangular harmonic analysis is adopted (Alldredge 1981). This approach is a part of a Cartesian coordinate, which is the solutions of Laplace's equation, and this approach fits to a planar rather than a sphere (De Santis *et al.* 1990). Finally, Haines in (Haines 1985) proposed the Spherical cap harmonic analysis (SCHA) model which overcomes geomagnetic field problems over a limited region of the globe by means of the SHA model. This SCHA technique is a prominent model in geomagnetic fields over a limited region of the globe and has more complex computations of Legendre functions with a non-integer harmonic degree (De Santis 1992). In this paper, the ASHA model is implemented for regional ionospheric modeling. The ASHA model is based on the extension of conventional spherical harmonics after the colatitude intervals are adjusted to the hemisphere (De Santis 1992). The Spherical Harmonic approach is based on the Fourier series expansion in terms of longitude-dependent and latitude-dependent associated Legendre function, which can be used to characterize the horizontal ionospheric profile (Opperman 2008). The orthogonality of spherical harmonic functions is derived from the expansion of the sphere; the associated Legendre functions $P_n^m(\cos\phi)$ are not orthogonal over the Earth's sphere but are lost within a certain area; the latitude dependent Legendre functions are orthogonal along the meridian when, $0 \le \theta \le 180°$ though two set of orthogonal functions in $0 \le \theta \le 90°$ to fit any general functions defined in this interval (De Santis 1991; De Santis *et al.* 1991).

2. Adjusted Spherical Harmonic Analysis

The orthogonality of the spherical harmonic function is derived from the expansion on the sphere, but it is lost within a certain area. Through coordinate conversion, the projection can be changed from a regional to global (An *et al.* 2014). First, the spherical cap coordinate system is constructed, in which the regional center has a new pole passing through the longitude line and the geographic South Pole is considered as the initial meridian. Then, the original coordinates are transformed to new coordinates. The Spherical cap pole coordinate system includes λ_0, φ_0 and the corresponding pierce point coordinates are λ_0, φ_0; therefore, λ_c, φ_c are the new pierce point spherical cap coordinates. As the selected area is India, which means that $\varphi_c = -\varphi$ and $\lambda_c = \lambda$. Assuming that θ_{max} is the spherical cap half-angle, the θ_c is the pierce point co-latitude of the spherical cap coordinate and its range is $[0, \theta_{max}]$, i.e., $\theta_c = \frac{\pi}{2} - \varphi_c$. The pierce point longitude remains the same and the latitude is projected onto an assumed sphere with a certain proportion according to the half-angle spherical cap. Thus, the new coordinates are as follows:

$$\begin{cases} \varphi' = \frac{\pi}{2} - \theta', \theta' = s.\theta_c \\ \lambda' = \lambda_c \end{cases}.$$

Co-latitude θ , then is adjusted to those θ' of the hypothetical hemisphere, with $s = 90/\theta_0$. (1)

The conventional Legendre functions will have the same properties with a non integer degree at the boundary intervals (see Fig. 1), with $n = k$. Equation (2) illustrates the mathematical model for the Vertical ionospheric delay by means of spherical harmonics.

$$TEC(\lambda, \theta') = \sum_{n=0}^{n} \sum_{m=0}^{m} \left(A_k^m \cos(m\lambda) + B_k^m \sin(m\lambda) \right) \times P_k^m(\cos\theta'). \quad (2)$$

Where λ indicates the longitudinal difference between the IPP and the sun direct spot;

θ' – adjusted colatitude with a hypothetical hemi sphere;

$P_k^m(\cos\theta')$ – normalized associated Legendre function;

A_k^m and B_k^m – unknown spherical harmonic coefficients.

With this conversion, the ionosphere pierce point coordinates are projected from the region to the assumed sphere, which meets the requirements for the fitting variables of the spherical harmonic function. Unknown spherical harmonic coefficients are computed by the parameter estimation method known as the Weighted least square regression method.

3. ASHA model advantages

The orthogonality of the spherical harmonic Analysis (SHA) is derived from the expansion of the sphere, but the Legendre functions are not orthogonal over the Earth's sphere. The ASHA has the advantage in regional modeling to overcome the limitations of the SHA. Through the coordinate conversion, the projection can be changed from a regional to global; errors are minimized and predict better estimation values.

3.1. Estimation of the Ionospheric Gradient

In equatorial and low latitude regions, the TEC has often been quite high with large spatial gradients. A Ionospheric Gradient method based on spherical harmonics is used for estimating the ionospheric gradient in East and North-South directions (Christie *et al.* 2000). The Ionospheric gradient in the East direction can be

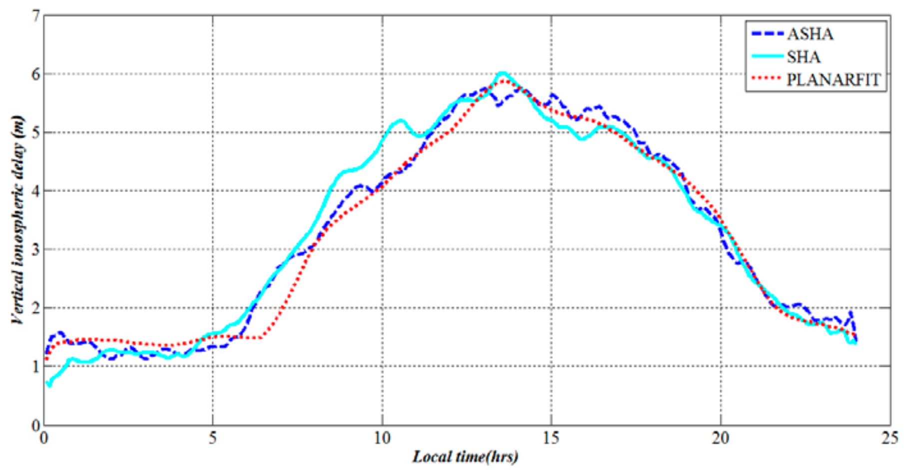

Fig.1. Estimated vertical delay variations on a typical quiet day (July 05, 2004) at IGP location (25°*N*, 75°*E*)

obtained by the differentiation of Equation (2) with respect to longitude (λ):

$$\frac{\partial TEC(\lambda,\theta^{'})}{\partial\lambda}=\frac{1}{R_e\cos\left(\frac{\pi}{2}-\theta^{'}\right)} \sum_{n=0}^{n}\sum_{m=0}^{m}\left(A_k^m\cos(m\lambda)+B_k^m\sin(m\lambda)\right)mP_k^m(\cos\theta^{'}). \tag{3}$$

Similarly, the gradient in the North-South direction can be obtained by the differentiation of Equation (4) with respect to the colatitude $(\theta^{'})$

$$\frac{\partial TEC(\lambda,\theta^{'})}{\partial\theta^{'}}=-\frac{1}{R_e}\sum_{n=0}^{n}\sum_{m=0}^{m}\left(A_k^m\cos(m\lambda)+B_k^m\sin(m\lambda)\right) \left\{\frac{m\times\cos\theta^{'}}{\sin\theta^{'}}P_k^m(\cos\theta^{'})-P_k^{m+1}(\cos\theta^{'})\right\}. \tag{4}$$

Spherical harmonic coefficients obtained from Equations (3) and (4) are used to calculate local ionospheric spatial gradients.

4. Results and discussions

Vertical TEC values are estimated based on the spherical harmonic model over the India region. These Vertical TEC values are estimated at the Ionospheric grid point (IGP) over the Indian region by taking measurements from 17 GPS stations, which are part of the GAGAN-TEC Network. The 3rd degree model was used for spherical harmonic coefficients. The ASHA model performance is evaluated by comparison with different grid models.

4.1. IGP delay estimation

4.1.1. Quiet day data

Here we consider July 05, 2004 as a magnetically quiet day for estimating the ionospheric delay over the region of India. The K_P index value on July 05, 2004 was in the range of $0<K_P<2$. The ASHA model can be evaluated by comparing with different grid models illustrated in Figure 1 at IGP (25° N, 75° E). It can be observed here that the vertical delay estimated by the ASHA is closely related to the Planar fit model and the SHA. Further comparison has been carried out at different IGPs and results of minimum and maximum delay are illustrated in Table 1.

4.1.2. Disturbed day data

Here we consider July 23, 2004 as a magnetically disturbed day to estimate the ionospheric delay over the region of India and the K_P index value on July 23, 2004 ranges within $0<K_P<6$. The ASHA model can be again evaluated by comparing with different grid models illustrated in Figure 2 at IGP (25° N, 75° E). Similarly, a further comparison at different IGPs is listed in Table 2. It is evident from the results that there are more vertical ionospheric delay variations for a disturbed day as compared to a quiet day.

4.2. Ionospheric Gradient towards the Eastern direction

Here again we selected July 05, 2004 (a magnetically quiet day) and July 23, 2004 (a magnetically disturbed day) for estimating the Ionospheric Gradient over the region of India. A Gradient method based on spherical harmonics is used for estimating the ionospheric gradient in the Eastern direction, as illustrated in Figure 3 at

Table 1. IGP delays observed at various IGP locations during the quiet day (July 05, 2004)

IGP delay Location	Ionospheric model	Minimum IGP delay		Maximum IGP delay	
		Meters	Hours(LT)	meters	Hours(LT)
$(25°N,75°E)$	SHA	0.664	0.1667	6.061	13.67
	ASHA	1.204	0.0833	5.734	13.83
	PLANARFIT	1.116	0.0833	5.872	13.58
$(25°N,85°E)$	SHA	1.293	0.75	6.03	14.5
	ASHA	1.351	0.0833	5.951	13.67
	PLANARFIT	1.116	0.0833	5.863	13.5
$(30°N,75°E)$	SHA	1.379	0.166	6.01	14.02
	ASHA	0.94	0.0833	5.869	12.83
	PLANARFIT	1.116	0.0833	5.872	13.58
$(30°N,85°E)$	SHA	1.476	0.1667	5.927	13.67
	ASHA	1.378	0.1667	5.83	13.83
	PLANARFIT	1.116	0.0833	5.87	13.67

Table 2. IGP delays observed at various IGP locations during the disturbed day (July 23, 2004).

IGP delay Location	Ionospheric model	Minimum IGP delay		Maximum IGP delay	
		Meters	Hours(LT)	meters	Hours(LT)
(25°*N*,75°*E*)	SHA	2.163	0.0833	8.627	16.58
	ASHA	1.47	0.0833	7.955	15.08
	PLANARFIT	1.911	0.0833	8.121	15.25
(25°*N*,85°*E*)	SHA	2.067	0.25	8.069	14.42
	ASHA	1.782	1.667	8.52	15.50
	PLANARFIT	1.911	0.0833	8.093	15.58
(30°*N*,75°*E*)	SHA	1.856	2.25	7.519	16.33
	ASHA	1.721	2.417	7.328	15.92
	PLANARFIT	2.028	0.1667	8.096	15.52
(30°*N*,85°*E*)	SHA	1.733	5.083	7.865	15.67
	ASHA	1.438	4.167	7.649	16.08
	PLANARFIT	1.686	5.417	8.097	15.67

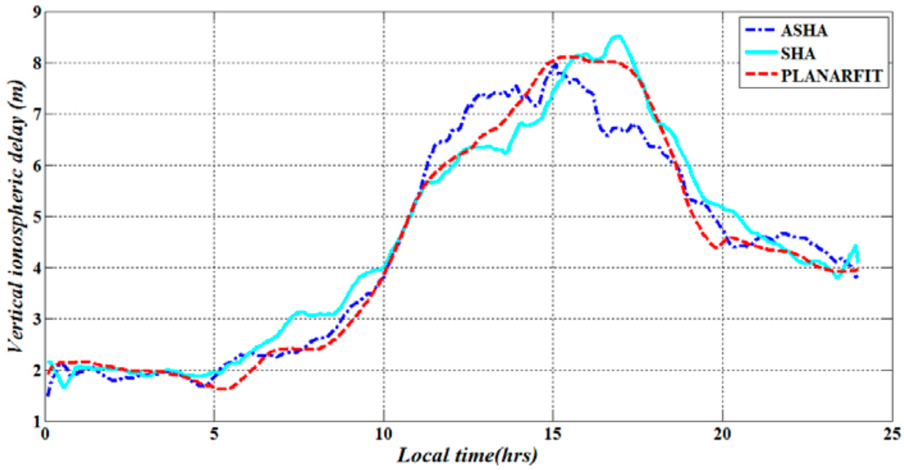

Fig. 2. Estimated vertical delay variations on a typical disturbed day (July 23, 2004) at IGP location (25°*N*, 75°*E*)

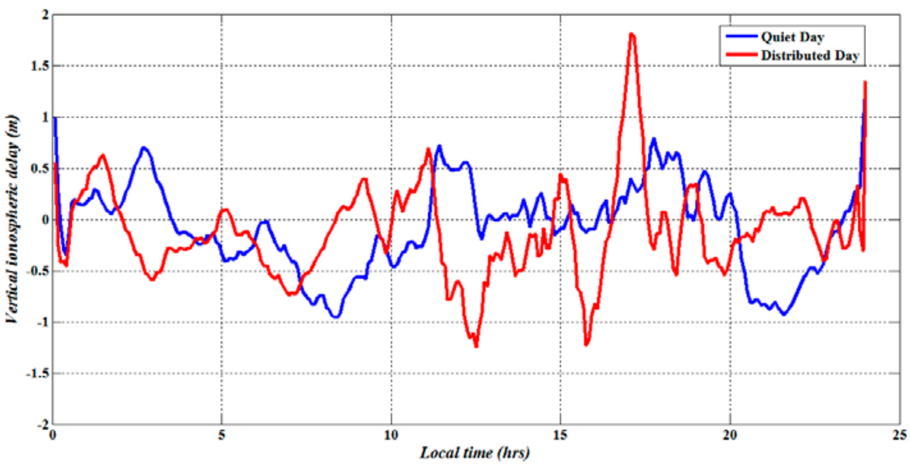

Fig. 3. Gradient of vertical Ionosphere delay towards East direction at IGP location (25°*N*, 75°*E*)

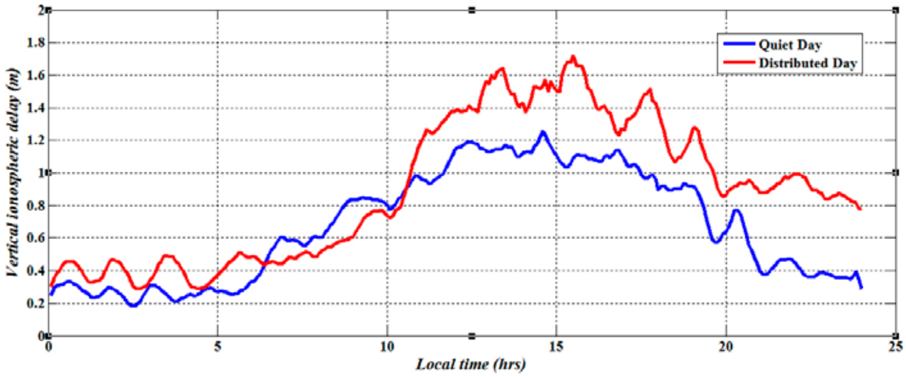

Fig. 4. Gradient of vertical Ionosphere delay towards North-South direction at IGP location (25°*N*, 75°*E*)

IGP (25° N, 75° E). It can be observed here that the maximum spatial gradient of vertical ionosphere is 1.3 m at 2400 hr and the minimum spatial gradient is –0.95 m at 8.41 hr for the quiet day. Similarly, for the disturbed day the maximum gradient is 1.80 m at 17.08 hr and the minimum gradient is –1.23 m at 15.75 hr.

4.3. Ionospheric Gradient towards the North-Southern direction

A similar exercise for the same days is carried out for estimating the Ionospheric Gradient towards the North-Southern direction over the region of India. A Gradient method based on spherical harmonics is used for estimating the ionospheric gradient in the North-Southern direction as illustrated in Figure 4 at IGP (25° N, 75° E). It can be noticed that contrary to the Eastern direction gradient, the North-Southern gradient is positive throughout the day. The maximum gradient is 1.25 m at 14.58 hr and the minimum is 0.18 m at 2.5 hr for the quiet day. Whereas, for the disturbed day, the maximum gradient is 1.71 m at 15.5 hr and the minimum gradient is 0.28 m at 2.58 hr.

5. Conclusions

A new methodology has been proposed to estimate the vertical delay at different Grid points using dual-frequency GPS measurements. The significance of the GPS-derived ionospheric TEC measurements for improving temporal variations with global and regional models was discussed. An adjusted spherical harmonic analysis based TEC model was implemented using the ionospheric delay observations over India and expressed in Sun-fixed longitude and adjusted co-latitude. The performance of the ASHA model is evaluated by comparison with different grid models. From the results it may be concluded that the ASHA model can also be considered for estimating ionospheric TEC variations from a regional network for satellite based navigation systems. The model can also be considered for estimating the ionospheric gradients to detect the irregularity in the ionosphere.

Acknowledgements

The present work of D. V. Ratnam was supported through F. 301/2013(SAII)/RA201416GEANP5585.

Contribution

The first author analyzed the 17 GPS-station data with the ASHA model with different grid models observed at IGP locations and drafted the manuscript. The second author identified the corrections while analysing the ASHA model with different grid models and guided the first author 1 in programming, drafting of the manuscript and verified the results. The third author verified the results and discussed with all the authors.

References

Alldredge, L. R. 1981. Rectangular harmonic analysis applied to the geomagnetic field, *Journal of Geophysical Research* 86(B4): 3021–3026. http://dx.doi.org/10.1029/JB086iB04p03021

An, J.; Wang, Z.; Ning, X. 2014. GPS-based regional ionospheric models and their suitability in Antarctica, 极地研究 25: 32–37.

Christie, J. R.; Ko, P. Y.; Hansen, A., *et al*. 2000. The effects of local ionospheric decorrelation on LAAS: theory and experimental results, *Ionosphere* 20: 0.

De Santis, A. 1991. Translated origin spherical cap harmonic analysis, *Geophysical Journal International* 106(1): 253–263. http://dx.doi.org/10.1111/j.1365-246X.1991.tb04615.x

De Santis, A. 1992. Conventional spherical harmonic analysis for regional modelling of the geomagnetic field, *Geophysics Research Letters* 19(10): 1065–1067. http://dx.doi.org/10.1029/92GL01068

De Santis, A.; Battelli, O.; Kerridge, D. J. 1990. Spherical cap harmonic analysis applied to regional field modelling for Italy, *Journal of Geomagnetism and Geoelectricity* 42(9): 1019–1036. http://dx.doi.org/10.5636/jgg.42.1019

De Santis, A.; De Franceschi, G.; Zolesi, B., *et al*. 1991. Regional mapping of the critical frequency of the F2 layer by spherical cap harmonic expansion, *Annales Geophysicae* 9(6): 401–406.

Dvinskikh, N. I.; Naidenova, N. Ya. 1991. An adaptable regional empirical ionospheric model, *Advances in Space Research* 11(10): 7–10. http://dx.doi.org/10.1016/0273-1177(91)90312-8

Haines, G. V. 1985. Spherical cap harmonic analysis, *Journal of Geophysical Research: Solid Earth (1978–2012)* 90(B3): 2583–2591.

Molina, F.; De Santis, A. 1987. Considerations and proposal for a best utilization of IGRF over areas including a geomagnetic observatory, *Physics of the Earth and Planetary Interiors* 48(3–4): 379–385.

Opperman, B. D. L. 2008. *Reconstructing ionospheric TEC over South Africa using signals from a Regional GPS network:* Doctoral dissertation. Rhodes University, Grahamstown, South Africa.

Ratnam, D. V. 2013. Estimation and analysis of user IPP delays using bilinear model for satellite-based augmented navigation systems, *Aviation* 17(2): 65–69. http://dx.doi.org/10.3846/16487788.2013.805864

Sunda, S.; Sridharan, R.; Vyas, B. M., *et al*. 2015. Satellite-based augmentation systems: a novel and cost-effective tool for ionospheric and space weather studies, *Space Weather* 13(1): 6–15. http://dx.doi.org/10.1002/2014SW001103

Zolesi, B.; Cander, Lj. R.; De Franceschi, G. 1993. Simplified ionospheric regional model (SIRM) for telecommunication applications, *Radio Science* 28(4): 603–612. http://dx.doi.org/10.1029/93RS00276