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MATHEMATICAL MODELING OF DATA TRANSMISSION SYSTEM IN COMMUNICATION NETWORKS

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Abstract. Probability failure in operation as well as error probability at fixed frequency band, number of users, and data transmission speed have been analyzed.

Keywords: probability failure, number of users, data transmission speed, communication systems, communication channels.

List of symbols:

t – transmission time of signal along channels
R_0 – modulation rate
ρ_k – code rate
n – length of data block
ε – index of error grouping, which depends on the channel types ($0 < \varepsilon < 1$)
ω – intensity of stable failure
T_B – average recovery time in system after failure
υ – intensity of stationary flow
$p_{\text{э}}$ – the probability of failure of a single element period
n_{opt} – optimal sense of length of data block
k – symbols information

1. Introduction

Mobile communication systems are intended to serve a great number of users [5, 8, *]. In order to achieve the operational target and increase effectiveness of such systems, space zone separation and free user access to the common frequency band have been provided [6].

2. Analysis of the problem

It is very important to determine quality criteria Q_0S of user service in mobile communication systems. For the determination of this criterion, it is necessary to take into account two aspects. On the one hand, mobile communication systems can be regarded as a data transmission system for which the quality criteria can be characterized by error probability when symbols at the fixed frequency band and

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data transmission speed are received. On the other hand, these systems are considered public mass service systems [1, 7].

The objective of this article is to apply mathematical modeling to the analysis of failure probability relationship versus data transmission speed [4]. The effective data transmission speed has been considered.
Applying the differential equation ($dR_e / dn = 0$), the value of n_{opt} has also been found (7).

The probability of operating failure, probability of failure at the fixed frequency band, number of users and information transmission speed in public mobile communication systems, in which the number of frequency bands and the load of the system connected with the data transmission speed can be calculated [2, 3]. The variants of the relationship of effective data transmission speed versus unit duration are presented in figure 1. The model $R_e = f(R_0, \rho_K, n, t, \varepsilon, \omega, T_B, v, p_e)$ can be instantiated having in mind connections among variables of the function $R_e = f(R_0, \rho_K, n, t, \varepsilon, \omega, T_B, v, p_e)$ [5, 8, *]. Let the data block contain K encoded symbols of n (i.e. $\rho_k = k / n$) and the probability of error determination while receiving data block is equal to p.

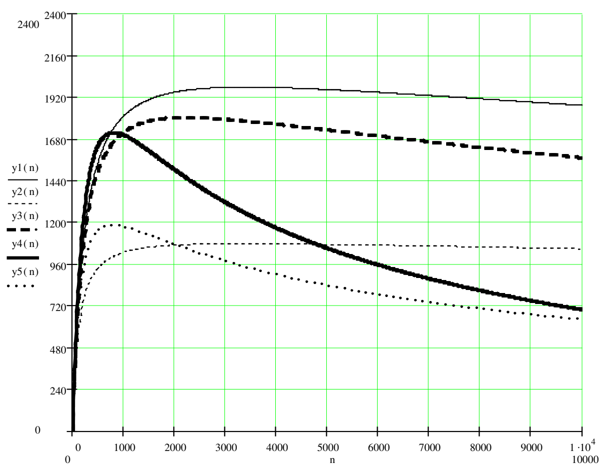


Fig 1. The relationships of $R_e / \rho_k = f(n)$

where R_0 – modulation rate;

ρ_k – code rate;

n – the length of data block;

ε – index of error grouping, which depends on the channel types ($0 < \varepsilon < 1$);

$\varepsilon = 0.3 \div 0.7$ is taken for cable and telephone channels; $\varepsilon = 0.4 \div 0.5$ is taken for radio relay transmission;

ω – intensity of stable failure, here $\omega = \upsilon / T_B$;

T_B – average recovery time in system after denial;

υ – intensity of stationary flow;

p_e – the probability of failure of a single element period.

Then a single transmission of data block requires the time described as

$$T = t + n / R_0, \qquad (1)$$

and the average time interval required for transmission of data block, taking into account the number of possible call repetitions, is expressed as

$$t_1 = T \sum_{i=0}^{v} p^i . \qquad (2)$$

Under these conditions, the probability of communication channel failure due to acceptable call repetitions v is equal to p^v and requires a recovery time period T_{B1} . During the time period t_1 with an interval equal to ω , if a hardware failure can occur, an average time period T_{B2} is required for its recovery. If $T_{B1} = T_{B2} = T_B$, the effective data transmission speed taking into account equations (1) and (2) is equal to

$$R_e = \rho_k \cdot n \left[T \sum_{i=0}^{v-1} p^i + p^v T_B + \omega T_B \cdot t_1 \right]^{-1} . \qquad (3)$$

As a result of the substitution of (1) and (2) into (3) and the terms regrouping for R_e we can get

$$R_e = \rho_k \cdot n \left[\left(t + \frac{n}{R_0} \right) \sum_{i=0}^{v-1} p^i + p^v \left(t + \frac{n}{R_0} + T_B \right) + \omega T_B \left(t + \frac{n}{R_0} \right) \sum_{i=0}^{v} p^i \right]^{-1} . \qquad (4)$$

If effective error detecting codes are applied in the communication systems, the following expression can be obtained

$$p \approx p(\geq 1, n) = p_e n^{\varepsilon} , \qquad (5)$$

where $p(\geq 1, n)$ is the probability of a failure in transmission of data block with the period of n , one or more elements; p_e – the probability of failure of a single element period; ε – index of error grouping, which depends on the channel types ($0 < \varepsilon < 1$).

After the substitution of (5) into (4) and not very complicated transformations, the expression for effective speed of data transmission is obtained:

$$R_e = \rho_k R_0 \frac{n(1 - p_0 n^{\varepsilon})}{(R_0 t + n) + \dfrac{\omega}{\beta}(R_0 t + n) + R_0 T_B \cdot p_e^{v} n^{v\varepsilon}} . \qquad (6)$$

where recovery is $\beta = 1 / T_B$.

The above expression clearly illustrates the influence of four main factors on the decrease of main R_e values. The multiplier in the numerator reflects the decrease in the value of R_e due to noise.

The first and the second summands in the denominator reflect the decrease in R_e due to hardware failure and the period of time required for recovery.

The third summand in the channel of communication is connected with the decrease in R_e due to a surge of noise in the channel and the limit growth of v number of call repetitions value.

Expression (4) for R_e in its nature can be regarded as an analytical model for the effective operation of a communication channel.

The analysis of this model (4) shows that R_e depending on n has the extreme type of maximum at $n = n_{opt}$, when optimum meaning of n_{opt} providing $R_{e\ max}$ can be determined from the following equation: $dR_e / dn = 0$. This equation, which has been differentiated for n, will result in the following transitional expression

$$n^{\varepsilon(v+1)} p_e^{v+1} R_0 T_B \left[\varepsilon(v-1)-1\right] + n^{\varepsilon v} p_e^{v} R_0 T_B (1-\varepsilon v) +$$

$$+ R_0 T_B \left(1+\frac{\omega}{\beta}\right) - n^{\varepsilon+1} p_e \varepsilon \left(1+\frac{\omega}{\beta}\right) - n^{\varepsilon} p_e R_0 t + \left(\frac{\omega}{\beta}+1\right) *$$

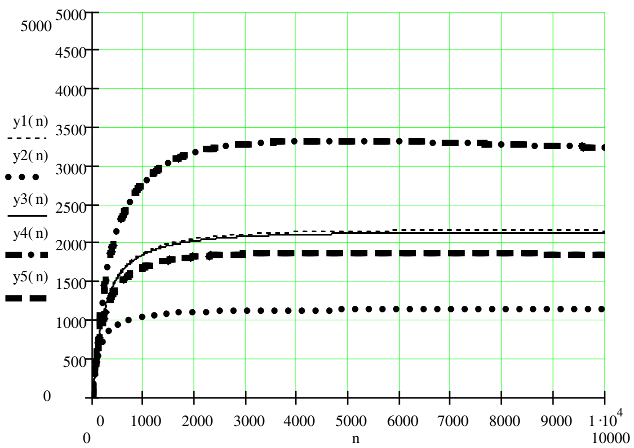
$$*(1+\varepsilon) = 0. \qquad (7)$$

As a general approach equation, (5) cannot have a finite analytical meaning relative to n. For some communication channels, when $\varepsilon, v, R_0, T_B, p_e$ and ω has definite values, it is possible to determine analytically the value of n_{opt}.

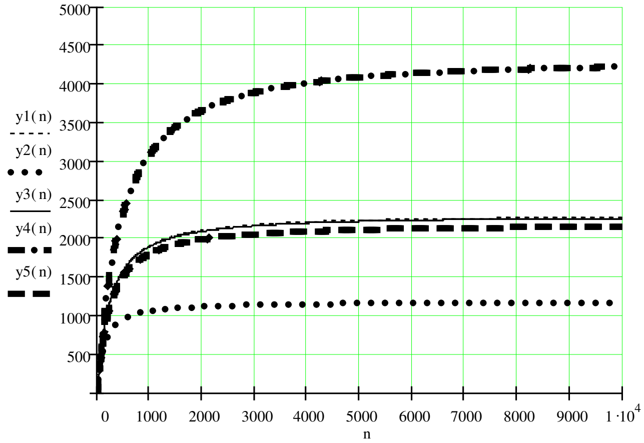
Examples for the relationships of $R_e = f(n)$ are shown in figure 1, where $\frac{R_e}{\rho_k}$ is replaced by $y_i(n)$

$$y_i(n) = \frac{R_e}{\rho_k}.$$

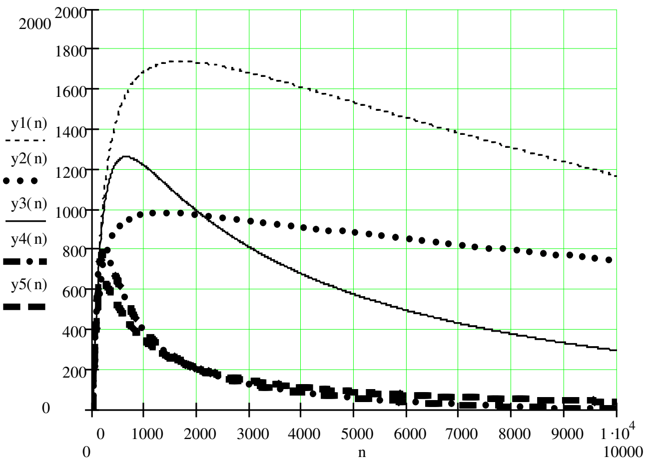
Now look at the different values of n with the change in ε pictured in figure 2.



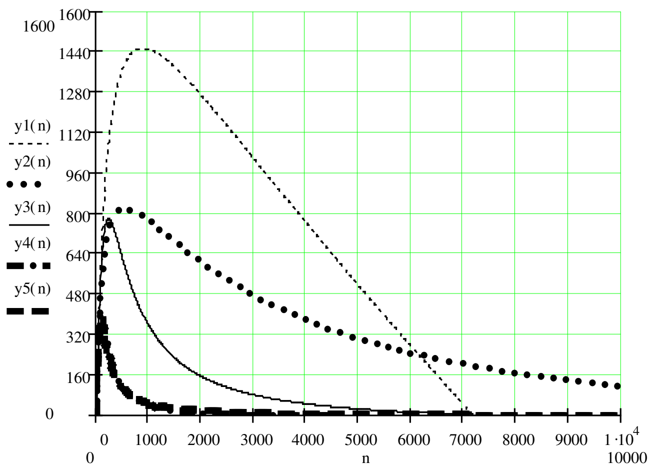
a) $\varepsilon = 0.3$



b) $\varepsilon = 0.4$



c) $\varepsilon = 0.6$



d) $\varepsilon = 0.7$

Fig 2. The relationships of different values of n with the change in ε

3. Reseach results

Results of the calculations carried out by means of (7) are written down as the relationship $R/\rho_k = f(n)$ and shown in figure 1. Curves 1, 2, and 3 are calculated for the following pairs of terms R_0 and p_e: R_0 = 1200 Baud, $p_e = 10^{-3}$; R_0 = 2400 Baud, $p_e = 4{\cdot}10^{-3}$; and R_0 = 4800 Baud, $p_e = 4{\cdot}10^{-3}$. The parameters $\nu = 3$, $\varepsilon = 0{,}5$, $T_B = 100$ s, t = 0.1s, and $\omega = 5 \cdot 10^{-5}\,\mathrm{h}^{-1}$ are considered identical for the aforementioned three curves.

Curves 4 and 5 are calculated for R_0 = 2400 Baud, $p_e = 2{\cdot}10^{-3}$, where t_0 has different values. Curve 4 is calculated at $T_B = 0$ (the ideal system recovery) and $T_B = 1000$ s has been chosen for curve 5. Thus, curves 4 and 5 taken together with curve 2 represent the family of relationships in which the varying term is $T_B = 0$, 100, 1000 s, which characterizes the recovery period of time.

4. Conclusions

1. Depending on the period of a unit of data, the effective speed of data transmitting R has a maximum value which at the increase of R_0 can be shifted to the side of smaller values of n_{opt}.
2. The extremum of relationship $R_e = f(n)$ is higher at larger values of R_0. This is evidence for the fact that with a decrease in the speed of data transmission, the critical importance of choice of n decreases.
3. Data transmission speed values R and n_{opt} drop with T_B growth (the system quality recovery goes down).
4. When the rate of error grouping in the channel becomes lower and the number of single errors grows (at ε growth), the value n_{opt} goes down, and it is not highly dependent on ε begining with values of ε = 0.5 … 0.6.
5. Values of n_{opt} can be essentially changed with the changes in p_e. Thus, if errors are independent (ε = 1), values of n_{opt} are changed by three orders of magnitude. If p_e is changed from 10^{-2} to 10^{-5}, the value of n_{opt} drops from 3,000 to 2,000 Baud (Fig 1).
6. T_B value greatly influences n_{opt}. If it changes from 0 to 1000, the value of n_{opt} goes down more than three times (from 300 to 100 Baud) (Fig 1; curves 2, 4 and 5).
7. The value of a permissible number of call repetitions ν (criterium of reliability versus nonstable failures) can be chosen in limits of $\nu = 3…5$ if these values of s are lower. The effective information speed essentially decreases, higher values of n_{opt} increase, and R goes up.

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DUOMENŲ PERDAVIMO RYŠIO TINKLAIS MATEMATINIS MODELIAVIMAS

W. Zijuan

S a n t r a u k a

Išnagrinėta aptarnavimo atmetimo tikimybė bei klaidų tikimybė fiksuotų dažnių juostose, abonentų skaičiaus ir informacijos perdavimo greičio nustatyme.

Reikšminiai žodžiai: avarijos tikimybė, naudotojų skaičius, duomenų perdavimo greitis, ryšio sistemos, ryšio kanalai.