

FLUTTER CALCULATION BASED ON GVT-RESULTS AND THEORETICAL MASS MODEL

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Abstract. Ground vibration tests (GVT) are a typical source of data for flutter prediction. In this paper, a simple, low-cost method to calculate flutter is presented. In this method, measured frequencies, mode shapes of an airplane are used and, additionally, the theoretical mass model of it.

If the theoretical mass model is used, it is possible to calculate generalized masses of modes and cross mass couplings between them. The mass couplings of normal modes should be zero. Orthogonalization is correction of the mode shapes to lead the couplings to zero. The possible orthogonalization methods are presented in chapter 2.

Based on eigenmodes of airplane configuration during GVT, it is possible to determine the eigenmodes of the same free airplane after a relatively small mass change, i.e. for another mass distribution that was not investigated by GVT. In the procedure presented in chapter 3, it is assumed that geometric and stiffness properties do not change.

The methodology was used in the own flutter calculation software that is useful for flutter prediction of light airplanes and sailplanes.

Keywords: aerodynamic flutter, ground vibration test, normal modes, mass model, orthogonalization, alteration of the mass distribution.

Nomenclature:

s – number of DOF of theoretical mass model of airplane

$[\mu]$ – mass matrix of dimension of $s \times s$ (origin mass distribution)

$[\mu']$ – mass matrix after variation of airplane mass distribution

n – number of measured modes of elastic airplane

N – number of displacement modes of airplane treated as a rigid body

$n+N$ – number of all modes of airplane

$\{\varphi_i\}$ – column matrix of i -th mode shape ($i = 1, \dots, n$) of elastic airplane (also called "elastic mode shapes")

$\{\varphi'_i\}$ – column matrix of i -th mode shape ($i = 1, \dots, n$) of elastic airplane after scaling or orthogonalization

$\{\psi_j\}$ – column matrix of j -th mode shape ($j = 1, \dots, N$) of airplane treated as a rigid body ("rigid mode shapes")

$[T] = [\{\varphi_1\}, \{\varphi_2\}, \dots, \{\varphi_n\}]$ – matrix of elastic mode shapes

$[R] = [\{\psi_1\}, \{\psi_2\}, \dots, \{\psi_N\}]$ – matrix of rigid mode shapes

- s – number of DOF of theoretical mass model of airplane
- $[Q] = [T | R]$ – matrix of all mode shapes of dimension of $s \times (n+N)$
- $[M]$ – modal mass matrix of dimension of $(n+N) \times (n+N)$
- $[M_{RR}] = [R]^T [\mu] [R]$ – rigid body mass matrix of dimension of $N \times N$
- $[K]$ – modal stiffness matrix of dimension of $(n+N) \times (n+N)$
- $[E]$ – projection matrix of dimension of $(n+N) \times n$
- $[I]$ – unitary matrix

1. Introduction

Aerodynamic flutter is an unsteady phenomenon with the interaction of mass, stiffness and aerodynamic forces. Methods to numerically predict flutter are often based on calculated or measured normal modes of a free airplane and simple or more advanced models of unsteady aerodynamics.

Classic methods of flutter analysis are linear and use normal modes as generalized DOFs. The flutter equation with this assumption can be transformed to an algebraic real or complex matrix equation of low range. The results of flutter calculation contain the critical flutter speed (for damping equal to zero), flutter frequency, and the flutter mode shape (as a complex column matrix) of existing flutter modes.

Ground vibration test (GVT) is a typical source of data for flutter prediction. At the beginning of classic GVT, the airplane to be tested is fitted with sensors (pick-ups) and supported by elastic suspension. To simulate the conditions of free flight, the highest normal frequency of the suspended airplane treated as a rigid body should be less than one-third of the lowest normal frequency of the normal mode, in which exists the structural elastic potential energy.

During classic GVT the structure is harmonically excited in several points. The frequency of the enforcement and the distribution of these forces are selected to drive the structure as near to resonance as possible. The frequency, the damping coefficient, the generalized mass, and the mode shape measured under resonance were treated as the successive normal vibration parameters of the structure. The GVT results are a complete set of the measured normal vibration with frequencies up to an upper frequency that can be determined based on maximal dive speed, length of aerodynamic chords, and the critical value of the Strouhal number.

The results of GVT are useful in the airplane design and certification processes:

1. To prove simplified flutter criteria NASA Report 45 (conservative and valid for conventional low-speed airplanes only).
2. To verify the theoretical computational FEM dynamic model by comparing measured and calculated normal vibration (the flutter calculation is provided based on the FEM-model, for example in the *Nastran*¹).

3. To perform flutter calculation in which the measured frequencies and mode shapes of normal vibrations are used directly.

The second method of flutter analysis is typical, but the last way is very attractive, especially for small airplanes, because it is credible, relatively rapid, and inexpensive (FEM-software is not necessary and the models are relative small and easy to prepare). In this case, the generalized masses can be measured or calculated based on theoretical mass model and measured mode shapes.

The presented method of flutter computation based on GVT results of an airplane and theoretical mass model of it. From our own experience, we know that when manually operating light airplanes and sailplanes this theoretical model of mass distribution should also contain the masses of most important parts of the control system: control wheels, sticks, push-rods, bob-weights, etc.

Measurements of generalized masses are not necessary, but these results can be used for comparison. Besides measured modes, theoretically generated rigid body modes and some unmeasured but existing control surface modes are taken into account.

In the next chapter, the orthogonalization methodology is presented. Orthogonalization is some correction of the measured mode shapes. The frequencies are not change.

To reduce costs, GVT is performed only for one or a few of the mass configurations of an airplane. Based on the eigenmodes of an airplane configured, as it would be for a GVT, it is possible to determine the eigenmodes of the same airplane after relative small mass changes, i.e. for another mass distribution that was not investigated by the GVT. In the procedure presented in chapter 3, it is assumed that geometric and stiffness properties do not change.

2. Orthogonalization of mode shapes

The possession of a theoretical mass model makes it possible to calculate the generalized masses of the mode defined above and the cross mass couplings between them:

$$[M] = [Q]^T [\mu] [Q].$$

The mass couplings matrix $[M]$ should be diagonal (i.e. normal modes should be $[\mu]$ - orthogonal), but as a result of measurement errors and errors in the theoretical mass model, it is generally full. Only the rigid body modes cannot be orthogonal, because its frequencies are the same (equal to zero).

For this reason, the measured mode shapes should be orthogonalized. Orthogonalization is some correction of

¹Trademarks are in *italics*:

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the mode shapes to lead the mass couplings to zero between measured modes.

For convenience, all mode shapes can be scaled:

$$\begin{aligned} \{\varphi_i'\} &= \{\varphi_i\} / (\{\varphi_i\}^T [\mu] \{\varphi_i\})^{1/2}, \quad i = 1, \dots, n; \\ \{\psi_j'\} &= \{\psi_j\} / (\{\psi_j\}^T [\mu] \{\psi_j\})^{1/2}, \quad j = 1, \dots, N, \end{aligned}$$

so after scaling the mass couplings matrix should be diagonal. The scaling will be the process before orthogonalization and also after each orthogonalization step.

Generally, here it is assumed that the mode shapes after orthogonalization, [Q], can be a superposition of all mode shapes before orthogonalization, [Q]:

$$\begin{aligned} [Q'] &= [Q] [G], \\ [Q'] \text{ should be } [\mu] \text{ - orthogonal, so} \\ [G]^T [M] [G] &= [I]. \end{aligned}$$

This expression that determines the [G] matrix is ambiguously even if we assume that it should be close to unitary matrix and mode shape changes should be as small as possible. It is necessary to make the auxiliary assumptions presented below. For each assumption exists a separate kind of orthogonalization. Mutual orthogonalization of rigid modes

For convenience we can select the rigid body modes, [R'], as [\mu] - orthogonal:

$$[R'] = [R] [U],$$

where [R] and [R'] denotes the rigid body vectors before and after orthogonalization and [U] contains the eigenvectors of the rigid body mass matrix:

$$[M_{RR}] = [R]^T [\mu] [R] = [U] \text{diag}(m_1, \dots, m_N) [U]^T.$$

Trivial orthogonalization

The simplest kind of orthogonalization is when the modal mass couplings are omitted. It is the normal practice when orthogonalization is not performed.

Orthogonalization with respect to other mode shapes

The orthogonalization of mode L with respect to another invariable mode m is done by this expression:

$$\{\varphi_L'\} = \{\varphi_L\} - \{\varphi_m\} (\{\varphi_m\}^T [\mu] \{\varphi_L\}).$$

This kind of orthogonalization should be used only for the orthogonalization of measured modes with respect to rigid body modes or other exact modes that will not be further orthogonalized.

Gram-Schmidt mutual orthogonalization

In this method of orthogonalization, the [G] matrix is determined from the Banachiewicz-Cholesky triangular decomposition of matrix $[M_{TT}] = [T]^T [\mu] [T]$. The sequence of vector modes selected for orthogonalization is significant because the first mode will be not changed, the second one is orthogonalized only with respect to the first, and so on.

Weighted proportional mutual orthogonalization

In weighted proportional orthogonalization, it is assumed that:

$$[G] = \text{diag}(\alpha_1, \dots, \alpha_L) [S],$$

where L is the number of mode shapes selected for this step of orthogonalization, α_i ($i = 1, \dots, L$) are weights, and [S] is a symmetric matrix, that:

$$[S] = (\text{diag}(\alpha_1, \dots, \alpha_L) [M_{TT}] (\text{diag}(\alpha_1, \dots, \alpha_L)))^{-1/2}$$

The main value of the square root of the matrix can be performed after a transformation of matrix [M_{TT}] to spectral form.

In practice, we use a few orthogonalization steps. In the first step, all measured modes are orthogonalized with respect to rigid body modes and often also with respect to selected control surface modes. Next we select the groups of the measured modes that should be orthogonalized mutually. Finally, all measured modes are orthogonalized by Gram-Schmidt mutual orthogonalization. We use the measured modes after orthogonalization in the flutter calculation as the normal modes of the investigated airplane.

Orthogonalization of measured modes can eliminate suspension errors and friction effects in the modes with deflection of control surfaces. The use of orthogonalization improves the flutter calculation results. In many cases, the critical flutter speed calculated with orthogonalization is lower than it is without orthogonalization.

3. Determination of normal modes of free airplane after mass changes

The normal modes of a free airplane after (relatively small) mass alterations can be determined based on its normal modes for original mass distribution. For this purpose, we assume that the small vibration of free airplane after mass variation can be expressed as:

$$\{x(t)\} = \{\varphi_i'\} e^{i\omega t} = [T | R] \{q\} e^{i\omega t},$$

where {q} is a column matrix of modal coordinates to be found, and [T] (dimension of $s \times n$) and [R] (dimension of $s \times N$) are matrices that contain (as columns) the [\mu] - orthonormal mode vectors of n normal vibration and N motion of the airplane treated as a rigid body. The matrices [T], [R] and [\mu] are known and refer to the airplane before the mass variation.

The solution {q} should satisfy the equation:

$$(-\omega^2 [M'] + [K]) \{q\} = 0,$$

with:

$$[M'] = [T | R]^T [\mu'] - \text{modal mass matrix after mass variation}$$

$$[K] = (2\pi)^2 \text{diag} - \text{modal stiffness matrix with N-dimensional singular subspace } (f_1^2, \dots, f_n^2, 0, \dots, 0) \text{ (this matrix is not changed)}$$

and condition of zero momentum in free flight:

$$[R]^T [\mu'] \{x\} = 0, \quad \text{i.e.} \quad [R]^T [\mu'] [T | R] \{q\} = 0.$$

To solve this equations we use the ideas of prof. Miron Nowak and obtain the eigenproblem of the symmetric, positively defined matrix $[E]^T [K] [E]$ of dimension $n \times n$:

$$\omega^2 \{q_D\} = [E]^T [K] [E] \{q_D\}$$

(ω^2 – eigenvalue, $\{q_D\}$ - eigenvector).

The normal frequencies and normal modes of the airplane after mass variation are:

$$f'_i = \omega_i / 2\pi, \quad \{\varphi'_i\} = [T | R] [E] \{q_{Di}\}.$$

The [E] projection matrix of the dimension of $(n+N) \times n$ contains as columns any orthonormal basis of space of the elastic modes of the airplane after mass change. The orthogonality in this case is defined as:

$$[E]^T [T | R]^T [\mu'] [T | R] [E] = [I].$$

To determine matrix [E], the columns of [T] matrix should be orthogonalized (with the mass matrix $[\mu']$, not $[\mu]$!) using an arbitrarily selected orthogonalization method mentioned in chapter 1. The [E] matrix can be found as a matrix that previously consists of the unitary matrix of the dimension of $n \times n$ and a null rectangular matrix of the dimension of $N \times n$. In this form, the matrix refers to situation before any mass alteration, i.e. $[\mu'] = [\mu]$ (the columns of [T] matrix are $[\mu]$ - orthonormal). Next the [E] matrix collects all changes of [T] matrix during the $[\mu']$ -orthogonalization of it, i.e. after mass change.

This method can be used to eliminate the influence of suspension masses added to an airplane during a GVT and to take into account the mass of fuel, changes of the mass balance of control surfaces, addition or change of external stores, etc.

A practical application of this methodology is the *POSTM.EXE* program

4. Conversion of GVT results and flutter calculation flow diagram

The mode shape measured under resonance should be transformed to calculate displacements connecting with the DOF of all elements of the mass model. It will also determine the displacements of aerodynamic elements.

Airplanes are usually symmetrical, so the basic assumption is the symmetry of the object of the vibration tests. The eigenmode shapes of ideally symmetrical objects are either symmetric or anti-symmetric. The mass model of a symmetrical airplane can contain the right (for example) half of structure.

To transform the GVT results to the required form, we use the *POSTR.EXE* program (see diagram below), which converts symmetrical and anti-symmetrical modes separately using symmetrization of the sensor indications. The sensors are located on both sides of the plane of symmetry of the tested object. That means that when the unsymmetrical vibration mode shape of the object has been received as the result of the test, both symmetrical and anti-symmetric parts may be isolated on its basis. This program interpolates the normal displacements and twist angles of the main airplane assemblies treated as beams (wing, each flap and aileron, fin, stabilizer, elevator, and rudder) along the span of it. It also determines the displacements connecting other DOFs of the mass model: the movement of wing/tail parts in its chord plane and displacement of stiff assemblies treated as a rigid body (engine, external stores, control wheel, stick).

For the purposes of the program, it has been assumed that during the vibration tests the airplane is fitted with one-axial sensors located approximately symmetrically with respect to the aircraft's plane of symmetry. The majority of the sensors should be localized so that they measure the vibration component normal to the lifting surface chord plane. The indications of it are used to determine aerodynamic forces and a portion of kinetic energy. The remaining sensors should be placed so that on the basis of their indications it is possible to determine all other degrees of freedom of the computational mass model.

The RYSR program can see the GVT results. This program has the capability of optionally correcting pick-up indications if the real measure direction of pick-up is not the same as nominal.

The transformation of GVT results to the form required by the flutter calculation system is provided by the *POSTR* program, which does the following:

- Symmetrisation of measured mode shapes (it is assumed that the airplane is symmetric, so its normal modes are either symmetric or anti-symmetric);
- Linear approximation of indications of pick-ups localized on successive section of wing, stabilizer, or control surface in flow direction;
- Interpolation of mode shapes along each element span using the spline functions;
- Calculation of mode shape vectors $\{\varphi_i\}$, $i = 1, \dots, n$ in the generalized coordinates compatible with the mass model, i.e. mass matrix $[\mu]$;
- Automatic creation of rigid body modes (translational and rotational modes of the airplane treated as a stiff body) $\{\psi_i\}$, $i = 1, \dots, N$. The rigid body vectors are defined as $[\mu]$ - orthogonal, (i.e. the matrix $\{\psi_i\}^T [\mu] \{\psi_j\}$ is diagonal) for convenience;
- Determination of generalized masses m_{ii} , modal mass matrix $m_{ij} = \{\varphi_i\}^T [\mu] \{\varphi_j\}$, and couplings of measured modes with rigid body modes, $m_{i(n+j)} = \{\varphi_i\}^T [\mu] \{\psi_j\}$, $i = 1, \dots, n, j = 1, \dots, N$;
- Optional, multi-step orthogonalization (see chapter 2) of measured modes (normal modes should be $[\mu]$ - orthogonal, i.e. the modal mass matrix should be diagonal).

The converted vibration modes are recorded in a *.z12 file, which is useful for *UBAHR* and *FLATH* programs. The *UBAHR* program records the *.wsi, *.wcz and *.wpo files used later to animate the vibration mode shapes (both normal modes and flutter modes) in the *RYSH.EXE* program. In this description * means any character string, and # means any natural number (in this case, a normal mode number). The names of "case.*" output files are automatically created on the basis of the file name *case.dpr*, which is the main data file for used in the *POSTR* program.

The *FLATH.EXE* program makes flutter calculations using classical aerodynamic *strip theory* and the *V-g* method of solving flutter equation. It uses the results of either *POSTR* or *POSTM* programs.

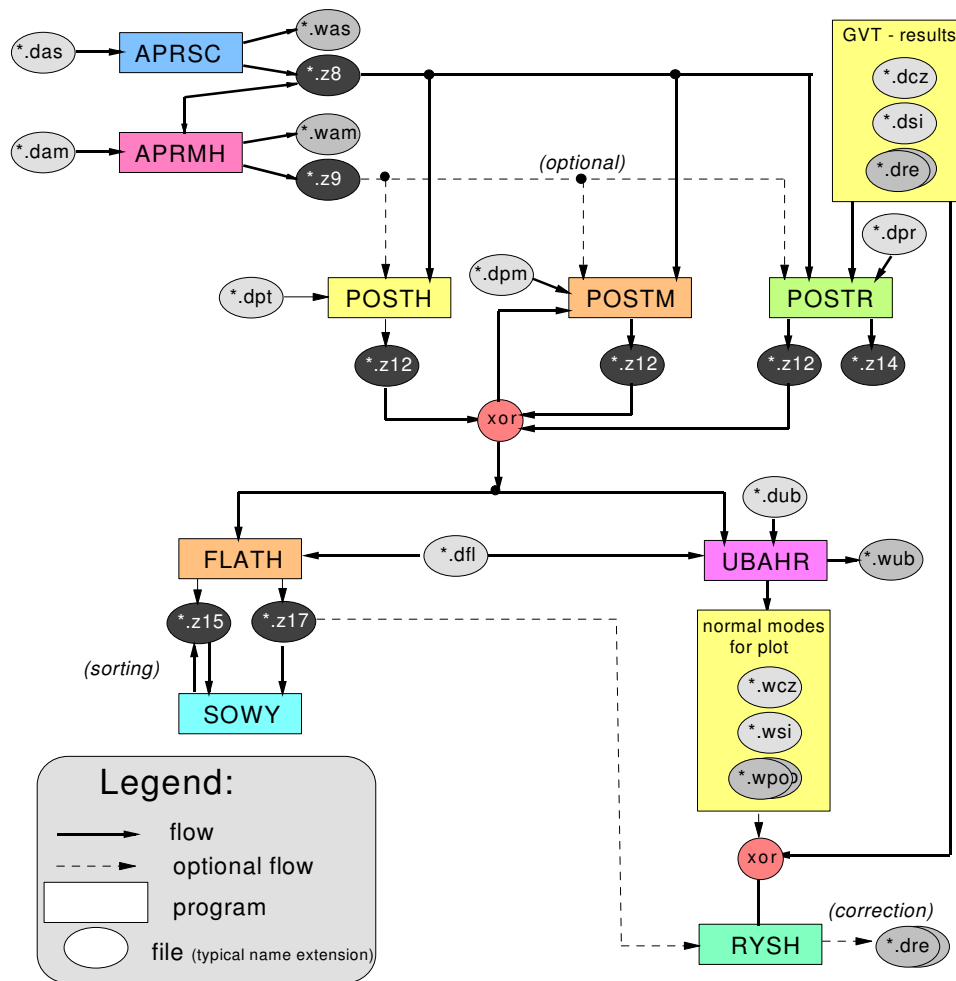


Fig 1. The flutter calculation flow diagram:

Programs used to calculate flutter based on GVT results (in alphabetical order):
APRSC.EXE – geometric definition and (not used here) stiffness data approximation – FORTRAN 77;
APRSM.EXE – approximation of mass distribution – FORTRAN 77. (This program is not often used because usually all mass data are defined as discrete masses);
FLATH.EXE – flutter calculation using Theodorsen’s strip aerodynamic theory for airfoil with (optional) control surface with (optional) tab, V-g method (K-method) of flutter equation solution, FORTRAN 77, *Microsoft Fortran Compiler, v.3.3* with library *flatippt.lib* in MS-Pascal and Assembler. A version compiled by the GNU Fortran Compiler also exists (Nowak 1972 ; Nowak *et al.* 1976; The ... 1976);
GVT2TAU.EXE – a program that contains the capacity of APRSC, POSTR, UBAHR programs and others – FORTRAN 77, GNU FORTRAN Compiler (Chajec 2004);
POSTH.EXE – natural vibration calculation based on geometry, stiffness, and mass distribution – FORTRAN 77. (For this purpose, we currently use *MSC.Nastran* rather than *POSTH.*) (Nowak *et al.* 1976; The ... 1976);

POSTM.EXE – natural vibration calculation based on (calculated or measured) natural vibration for alternate mass model (after a small alteration of mass distribution) of the same airplane (stiffness data without change), see chapter 3 – FORTRAN 77 (Chajec 1989);
POSTR.EXE – natural vibration calculation based on results of ground vibration test (GVT) and theoretical model of mass distribution; with orthogonality proof and, if necessary, optional orthogonalization of measured modes (Baruch 1980; Chajec 1980-1982; Chajec *et al.* 1985; Chajec 1989); three methods of orthogonalization are possible; see chapter 2–FORTRAN 77. –*Turbo Pascal v.6*. A similar program for the proof, correction, animation and plot of GVT results has the name *RYSR.EXE*;
SOWY.EXE – ordering of flutter results (also from *.f06–file from *MSC.NASTRAN*), damping versus speed V, and frequency versus V diagrams with optional vectors of flutter modes draw; V can be true (*TAS*) or equivalent (*EAS*) air speed; *Turbo Pascal v.6*;
UBAHR.EXE – vibration mode calculation postprocessor – FORTRAN 77.

The programs *APRSC.EXE*, *POSTR.EXE* and *FLATH.EXE* (included in the *JG2-system*) were made by a group led by prof. Miron Nowak in IPPT PAN, Warszawa (Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw) with some modifications made in PZL-Mielec (Chajec 1999).

The *APRSC*, *POSTR* and *UBAHR* programs were included in the program *GVT2TAU.EXE* (Chajec 2004).

Data file type (more important, in alphabetical order):

- *.*das* – geometry and stiffness data for *APRSC.EXE*-text file;
- *.*dgt* – text data for *GVT2TAU.EXE*. This data includes *.*das*, *.*dpr*, *.*dub* and *.*dfl* data files;
- *.*dfl* – text data for *FLATH.EXE* and *UBAHR.EXE* (geometry of aerodynamic surfaces, selected numbers of natural vibration);
- *.*dpm* – text data for *POSTM.EXE* (containing data of concentrated masses);
- *.*dpr* – text data for *POSTR.EXE* (data of concentrated masses, geometry of pick-ups/sensors/in ground vibration test, orthogonalization data);
- *.*dsl* – topology (connection for mode shape drawing) of pick-ups/sensors/in GVTtext file;
- *.*dcz* – coordinates of pick-ups in GVT – text file;
- *.*#dre* – indications (real and imaginary parts of accelerations similar to displacements) of pick-ups– for each measured modes numbered # one text file;
- *.*dub* – complementary data for *UBAHR.EXE* (auxiliary very small text file);
- *.*wsl* – topology (connection for mode shape drawing) of plot nodes – text file, result of *UBAHR.EXE*;
- *.*wcz* – coordinates of plot nodes – text file, result of *UBAHR.EXE*;
- *.*#wpo* – displacements of plot nodes in calculated vibration modes – text files, results of *UBAHR.EXE*;
- *.*wub* – synthetic results of *UBAHR.EXE*: frequency, modal damping, and generalized mass for each natural vibration mode – text file;
- *.*z8* – geometry and stiffness database (result of *APRSC.EXE*) – text file;
- *.*z12* – interpolated results of natural vibration calculation – binary file;
- *.*z14* – modal mass matrices before, during and after orthogonalization – text files;
- *.*z15* – unsorted or sorted results of flutter calculation (text file, with each record containing: reduced frequency as a parameter for the K-method, and for every flutter eigenvalue: speed, frequency and damping);
- *.*z17* – results of flutter calculation-flutter modes – text file.

5. Application

We use this low cost flutter calculation system to carry out the flutter calculation of general aviation airplanes, ultralight airplanes, and sailplane. Normally we use the simple classic V-g flutter calculation method with Theodorsen's strip aerodynamic theory. In this case, the calculations are very rapid and inexpensive.

These methods were used as the main methods in the EASA-certification of the I-23 airplane, in the preliminary flutter calculation of the M 28 Skytruck (the certification of this airplane was based on flutter calculations in the *MSC. Nastran* system), and to receive the permit for in-flight flutter tests of the following: EM-11C Orka (both versions: classic and *T* – tailplane), M21 Dromader Mini, M24 Dromader Super airplanes and SZD-55, PW-5 Smyk, Swift S-1, MDM-1 Fox sailplanes. It was also used in the flutter investigation of the JK-05 Junior ultralight.

As an example, the flutter calculation results of the I-23 carbon/glass composite piston airplane for four persons are presented here. The flutter mode shape *V* is shown on the next page.

The critical flutter speed was increased as a result of moving the balancing mass from rudder root to rudder tip.

It is also possible to use the measured (and numerically transformed and improved) modes to calculate the dynamics of a free, elastic airplane with more advanced unsteady aerodynamic models, including the 3D fluid mechanics method. For example, the transformed GVT results of the *I-23* airplane were used in *TAURUS*, and *CESAR* European projects with 3D unsteady aerodynamic models (Chajec 2004).

6. Conclusion

The classic methods of flutter analysis are based on theoretical geometry, mass, and stiffness model. In this case, it is possible to take into account in the flutter calculation any changes in the aircraft. The model should be adequate for real structures, so construction of them is very time consuming and expensive.

The calculation of flutter based on GVT results is more realistic, relatively inexpensive, and robust, but concerns only the tested configuration of an airplane.

The flutter calculation method presented here provides the possibility to numerically take into account in the GVT results the variation of the mass distribution of the airplane being tested. The method is cost efficient, consumes relatively little time, and can be used in practice to certify light aircraft and sailplane. The mass alteration magnitude is limited by the modal approach that was used.

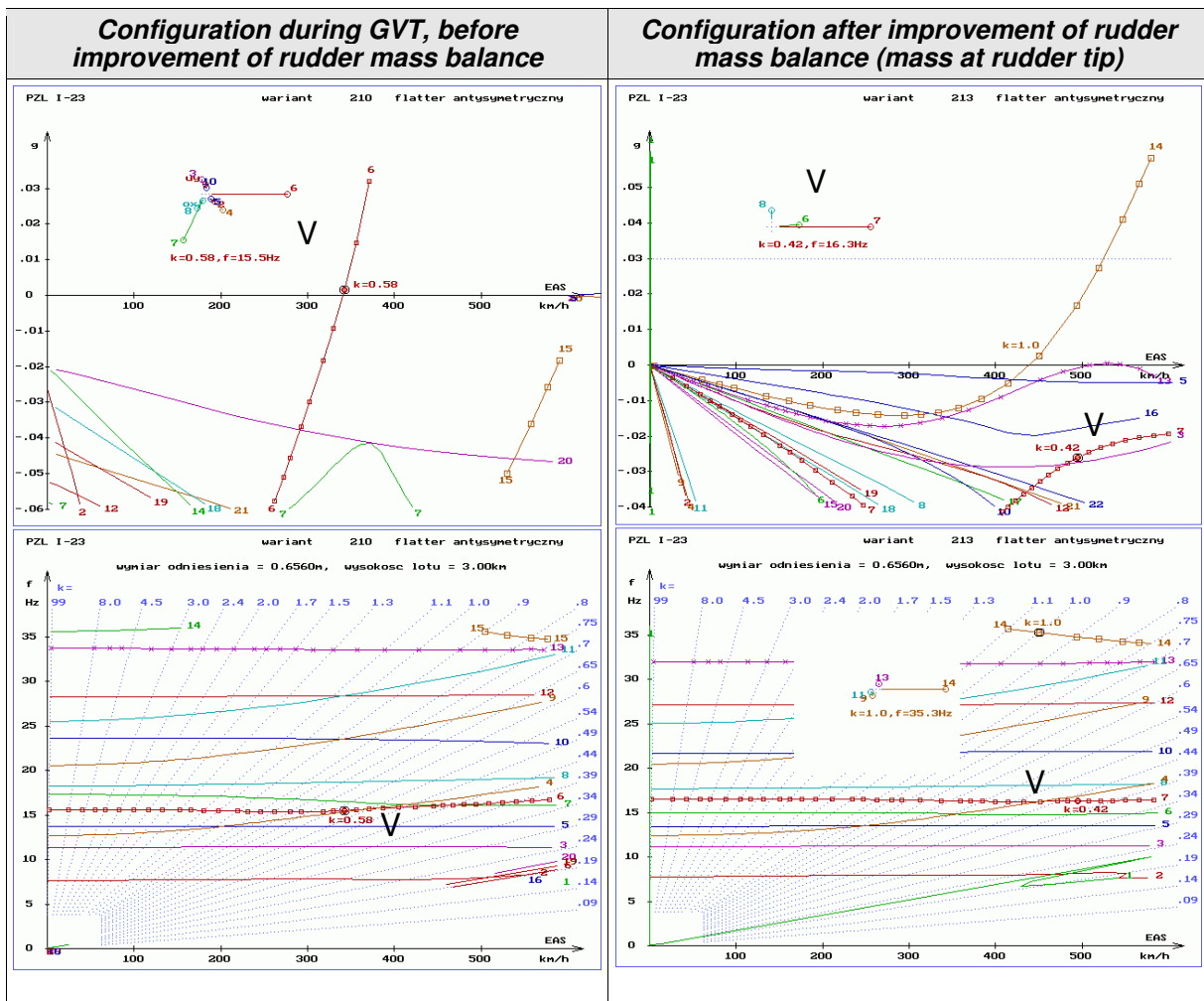


Fig 2. Results of anti-symmetric flutter calculation of I-23 airplane. The modal structural damping coefficients were not included

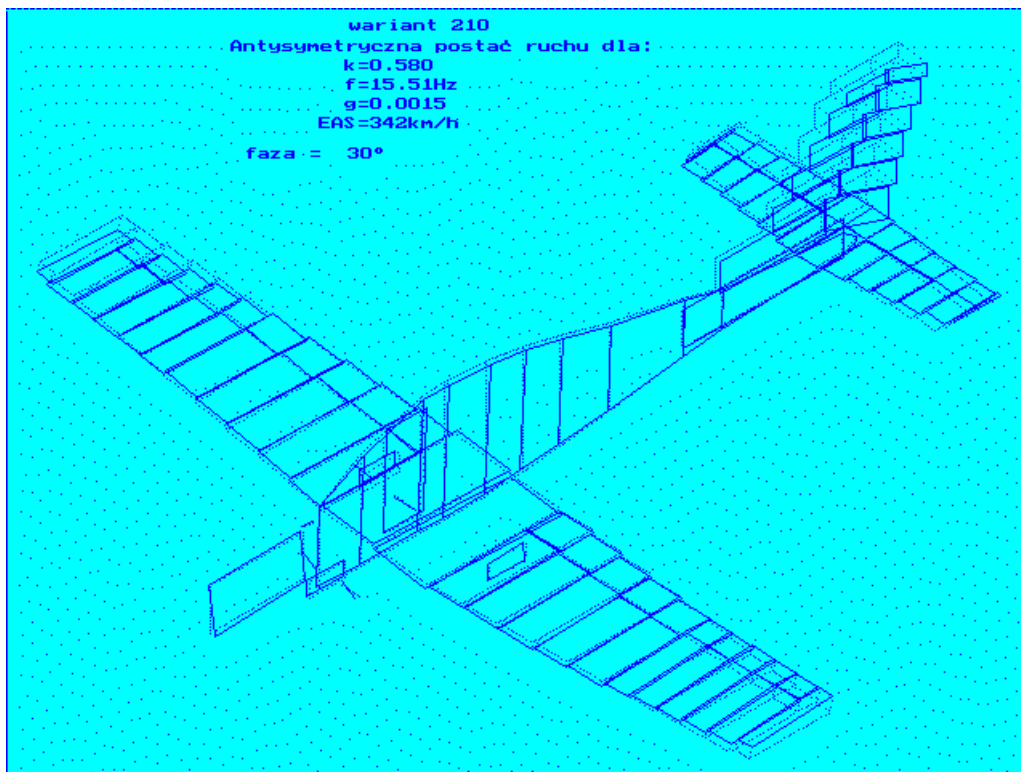


Fig 3. The V flutter mode before improvement of rudder mass balance

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FLATERIO SKAIČIAVIMAS REMIANTIS DAŽNUMINIŲ BANDYMŲ REZULTATAIS IR TEORINIŲ MASIŲ MODELIU

W. Chajec

Santrauka

Dažnuminiai bandymai žemėje yra tipinis informacijos šaltinis flaterio skaičiavimui. Šiame straipsnyje pateikiamas paprastas ir pigus flaterio skaičiavimo metodas. Šiame metode naudojamos lėktuvo išmatuotų dažnuminių modų formos ir teorinis lėktuvo masių modelis. Naudojant teorinį masių modelį galima apskaičiuoti apibendrintas modų mases ir masių ryšius tarp jų. Normalinių modų masių ryšys turi būti lygus nuliui. Ortogonalizavimu koreguojamos modų formos, siekiant ryšius sumažinti iki nulio. Galimi ortogonalizavimo metodai pateikti antrame skyriuje. Remiantis lėktuvo laisvųjų svyravimo modomis, gautomis dažnuminių bandymų žemėje metu, galima nustatyti kitokio masių pasiskirstymo įtaką laisvųjų svyravimų modoms. Procedūroje, pateiktoje trečiame skyriuje, manoma, kad geometrinės ir standumo savybės nesikeičia. Ši metodologija buvo panaudota savoje programinėje įrangoje flateriui skaičiuoti, kurią galima naudoti lengvų lėktuvų ir sklandytuvų flaterio skaičiavimui.

Reikšminiai žodžiai: aerodinaminis flateris, dažnuminiai bandymai žemėje, normalinės modos, masių modelis, ortogonalizavimas, masių išsidėstymo pokytis.