IDENTIFICATION OF ELASTIC PROPERTIES OF STIFFENED COMPOSITE SHELL

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Abstract. In the present study a numerical-experimental method for the identification of the elastic properties of stiffened composite panel from experimental vibration analysis is carried out. The elastic properties of a panel are found by an identification procedure based on the method of experiment design, the response surface approach, and the finite element method. The results obtained were verified by comparing the experimentally measured eigenfrequencies with the numerical ones obtained by FEM at the point of optima.

Keywords: finite element model, identification, response surface, vibration test, POLYTEC.

1. Introduction

Composite laminates are used extensively in the aerospace industry, especially for the fabrication of highperformance structures. The determination of stiffness parameters for complex materials, such as fibrereinforced composites, is much more complicated than for isotropic materials. A conventional way is testing the coupon specimens, which are manufactured by technology similar to that used for the real, large structures. When such a method is used, the question arises of whether the material properties obtained from the coupon tests are the same as those in the large structure. Therefore, the determination of actual material properties for composite laminates using non-destructive evaluation techniques has been widely investigated.

A number of various non-destructive evaluation techniques have been proposed for determining the material properties of composite laminates (Bar-Cohen 1986; Structural... 1997; Farrar *et al.* 2001). In the present study, attention is focused on the identification of the elastic properties of stiffened laminated panels with

one stringer using vibration test data. Vibration testing of a modal is a rapid and inexpensive method to obtain data to identify of elastic properties (Gibson 2000). There is a great deal of information in the literature on the identification of the elastic constants of laminated plates employing vibration test data (Pedersen, P. 1989; Mota Soares et al. 1993; Grediac et al. 1993; Moussu et al 1993; Frederiksen 1997a, b, c; Araújo et al. 2000; Hongxing et al. 2000). The problem associated with vibration testing is converting the measured modal frequencies to elastic constants. A standard method for solving this problem is the use of a numericalexperimental model and optimization techniques (Pedersen, P. 1989; Mota Soares et al. 1993; Frederiksen 1997a, b, c; Araújo et al. 2000). The identification functional represents the gap between the numerical model response and the experimental one. This gap should be minimized, taking into account the side constraints on the design variables (elastic constants). The minimization problem was solved by using nonlinear mathematical programming techniques and sensitivity analysis (Mota Soares et al. 1993; Frederiksen 1997a, b, c; Araújo et al. 2000). A similar identification functional has been employed in (Rikards et al. 1999; Rikards et al. 2001), but the minimization method was different. Instead of the direct minimization of the functional, the experiment design and response surface approach are employed for approximation of the numerical (finite element) model. Such an approach can reduce the computational efforts significantly.

In order to reduce the computational efforts, methods based on the approximation concepts were used in the structural optimization for the first time (Barthelemy et al. 1993.) The development of approximation functions has become a separate problem in optimum structural design (Toropov 1989). Approximating models can be built in different ways. The empirical model building theory is discussed by G. E. P. Box and N. R. Draper (Box et al. 1987). To construct a more general model of the original function, the methods of experiment design (Audze et al. 1977; Haftka et al. 1998) and approximate model building (Eglais 1981; Meyer et al. 1995; Khuri et al. 1996) can be used. A simplified model, called a meta-model (Auzins et al. 2002), is elaborated using results of the numerical experiment at a sample point of the experiment design. The response analysis using the simplified model is computationally much cheaper than the solution employing the original model. Despite the great variety of literature available on the identification of the elastic constants of laminated plates, there are very few studies dedicated to the problems of the estimation of the elastic parameters of stiffened plates. By S. Chakraborty and M. Mukhopadhyay (Chakraborty et al. 2000), the determination of in-plane elastic constants of stiffened plates was performed. In that study, instead of a physical experiment, numerical vibration data were used to determine the elastic constants.

In the present study, a numerical-experimental method to identify the elastic properties of stiffened composite panel from experimental vibration analysis has been developed. The elastic properties of a stiffened composite panel are found by an identification procedure based on the method of experiment design, the response surface approach, and the finite element method. The results obtained were verified by comparing the experimentally measured eigenfrequencies with the numerical ones obtained by FEM at the point of optima.

2. Parameters of identification

The proposed numerical-experimental approach is used to identify the elastic properties of a stiffened composite panel. A panel with one stringer was cut out from an original three-stringer panel. The dimensions of this panel are the following: length 290 mm, width 139 mm, and rib height 14.8 mm (Fig. 1).

The parameters to be identified are the elastic constants of a single layer in the stiffened composite panel with one stringer. These five parameters of a transverse isotropic layer are

- two Young's modulus: $E_1, E_2 = E_3$;
- Shear modulus: $G_{12} = G_{13}$.



Fig 1. One-stringer composite panel

Since, for a one-stringer composite panel, some elastic constants are less sensitive to frequencies, two of the five independent elastic constants are fixed (Rikards *et al.* 2003):

- Shear modulus: $G_{23} = 6.0$ GPa;
- Poisson's ratio: $v_{12} = 0.34$.

Thus, the identification of only three elastic constants $x = (E_1, E_2, G_{12})$ of the single layer is carried out.

3. Numerical-experimental method

The numerical-experimental method proposed in the present investigation consists of the experimental set-up, numerical model, and material identification procedure (Fig. 2).



Fig 2. Identification procedure

In the first stage, the finite element model of a stiffened composite panel has been used to model the frequency response of the structure with the initial guess values of elastic constants. The initial guess values of elastic constants have been determined by using the method of planning of experiments. In the second stage, these numerical data are taken to determine simple functions using response surface methodology. In parallel, vibration experiments are carried out with the purpose of determining the natural frequencies of the structures. In the third stage, identification of material properties is performed in the final stage, minimizing the error functional between the experimental and numerical parameters of structural responses.

3.1. Plan of experiments

Let us consider a criterion for the elaboration of the plans of the experiment. This criterion is to be independent of the mathematical model of the designed object (Rikards 1993). The initial information for development of the plan is the number of factors n and the number of experiments k. The points of experiments in the domain of factors are distributed as regularly as possible (Figs. 3, 4). For this reason the following criterion is used:

$$\Phi = \sum_{i=1}^{k} \sum_{j=i+1,\dots,n}^{k} \frac{1}{l_{ij}^2} \Longrightarrow \min, \qquad (1)$$

where l_{ij} is a distance between the points having numbers *i* and *j* ($i \neq j$). Physically it is equal to the minimum of potential energy of repulsive forces for the points with unity mass if the magnitude of these repulsive forces is inversely proportional to the distance between the points.



Fig 3. Plan of experiment: 2D view

The matrix of plan Bij characterizes the plan of the experiment. The domain of the experiments is determined as $x_j \in [x_j^{\min}; x_j^{\max}]$ and the points of the experiments are calculated by the following expression:

$$x_{j}^{(i)} = x_{j}^{\min} + \frac{1}{k-1} \left(x_{j}^{\max} - x_{j}^{\min} \right) \left(B_{ij} - 1 \right)$$
(2)

Here i = 1, 2, ..., k and j = 1, 2, ..., n.



Fig 4. Plan of experiment: 3D-view

3.2. Finite element analysis

The finite element solution is performed employing the commercial finite element software ANSYS 11.0. The finite element model of a one-stringer composite panel is modelled by using 1700 layered eight-node shear-deformable shell elements. Each node (5370) has six degrees of freedom, namely three displacements and three rotations.

The stiffened composite panel with one stringer is modelled with SHELL 99 linear layered, structural shell elements. The finite element model is presented in figures 5 and 6.



Fig 5. Finite element model of the panel

For the skin, [+45/-45/0] laminate is considered. The ply thickness t = 0.125 mm is fixed due to the manufacturing technology. Therefore, the thickness of the skin is h = 0.75 mm. The laminate lay-up for the blade-type stringer is $[(+45/-45)_3/0_6]_2$, i.e., the stringer consists of 24 single layers, and the thickness of the stringer is $b_{\rm w}$ = 3 mm. The stringer flange is stepwise flattened for a better matching with the contour of the skin. The stringer flange consists of three steps: the inner flange steplaminate stacking sequence [+45/-45]₃, i.e., six layers with the thickness $h_i = 0.75$ mm, the middle flange steplaminate stacking sequence [+45/-45]₂, i.e., four layers with +{the thickness} $h_{\rm m}$ = 0.5 mm, and the outer flange step-laminate stacking sequence [+45/-45], i.e., two layers with +{the thickness} $h_0 = 0.25$ mm. The density of the panels, as measured by hydraulic weighting, is $\rho = 1560.9 \, (\text{kg/m}^3).$



Fig 6. Finite element model with zoom for stringer

3.3. Response surface method

At the time of the present study, the form of the equation of regression was previously unknown (Eglais 1981). There are two requirements for the equation of regression: accuracy and reliability. Accuracy is characterised as a minimum of standard deviation of the table data from the values given by the equation of regression. With an increase in the number of terms in the equation, it is possible to obtain complete agreement between the table data and the values given. However, it is necessary to note that prediction at the intervals between the table points can be poor.

For an improvement in prediction, it is necessary to decrease the distance between the points of experiments by increasing the number of experiments or by decreasing the domain of factors. The reliability of the equation of regression can be characterized by an affirmation that standard deviations for the table points and for any other points are approximately the same. Reliability is obvioussly greater for a smaller number of terms in the equation of regression.

The equation of regression can be written in the following form:

$$y = \sum_{i=1}^{p} A_i f_i(x_j),$$
 (3),

where A_i are the coefficients of the equation of regression, $f_i(x_j)$ are the functions from the bank of simple functions $\theta_1, \theta_2, \dots, \theta_m$ which are assumed as:

$$\boldsymbol{\theta}_m(\boldsymbol{x}_j) = \prod_{i=1}^s \boldsymbol{x}_j^{\alpha_{ki}} , \qquad (4)$$

where α_{ki} is a positive or negative integer including zero and *s* is the number of object parameters. Synthesis of the equation from the bank of simple functions is carried out in two stages: selection of prospective functions from the bank and then step-by-step elimination of the selected functions.

In the first stage, all variants are tested with the least square method, and the function which leads to the minimum of the sum of deviations is chosen for each variant. In the second stage, the elimination is carried out using the standard deviation

$$\sigma_{0} = \sqrt{\frac{S}{k-p+1}} , \ \sigma = \sqrt{\frac{1}{k-1} \sum_{i=1}^{k} \left(y_{i} - \frac{1}{k} \sum_{j=1}^{k} y_{j} \right)^{2}}, \ (5)$$

or correlation coefficient

$$c = \left(1 - \frac{\sigma}{\sigma_0}\right) * 100\% , \qquad (6)$$

where k is the number of experimental points, p is the number of selected prospective functions, and S is the minimum sum of deviations. It is more convenient to characterize the accuracy of the equation of regression by the correlation coefficient (Fig. 11).

If insignificant functions are eliminated from the equation of regression, the reduction of the correlation coefficient is negligible. If in the equation of regression only significant functions are presented, the elimination of one of them leads to an important decrease in the correlation coefficient.

The method examined above for the approximation of the table data was the basis for the creation of the

program RESINT, which was used in this research (Rikards 1993).



3.4. Vibration experiment set-up

The stiffened composite panel with one stringer was tested for vibration in order to measure the eigenfrequencies and the corresponding modes. A POLYTEC PSV-400-B Scanning Laser Vibrometer measured the natural frequencies of the test panels. The equipment consisted of a PSV-I-400 LR optical scanning head equipped with high sensitivity vibrometer sensor (OFV-505), an OFV-5000 controller, a PSV-E-400 junction box, a Bruel & Kjaer type 2732 amplifier, and a computer system with data acquisition board and PSV Software. The system requires defining the geometry of the object and setting up a scanning grid. One hundred thirty points were taken to cover a rectangular panel with a regular grid. A macro-fibre composite actuator excited the test panel. After the measurement was performed at one point, the vibrometer automatically moved the laser beam to another point at the scan grid, measured the response using the Doppler principle, and validated the measurement with the signal-to-noise ratio. The procedure was repeated until all scan points were measured. The frequency spectrum of the panel was then obtained by taking the Fast Fourier Transform of the response signal.



Fig 7. Experiment set-up with MFC



Fig 8. A stiffened composite panel with MFC

Figure 7 shows the experimental set-up for the modal test of the stiffened composite panel with one stringer. Free-free boundary conditions were simulated by hanging the panel with two thin threads bonded in the two top corners of the plate. The MFC actuator was attached to the surface of the plate at the left corner of the bottom edge (Fig. 8).

3.5. Minimization of error function

The identification process was carried out through the minimization of the error function that expresses the relative difference between measured f_i^{exp} and numerically calculated eigenfrequencies f_i^{FEM} .

$$\Phi_i(\mathbf{x}) = \sum_{i=1}^m \frac{\left(f_i^{\exp} - f_i^{FEM}\right)^2}{\left(f_i^{\exp}\right)^2} \Longrightarrow \min$$
(7)

4. Identification and verification of results

The program EDAOpt was used for the generation of the plan of the experiment (Janushevskis *et al.* 2004). The plan of the experiment was formulated for five design parameters and 101 experiments (Figs. 3, 4). The upper and lower limits of the identification parameters were taken as follows:

$$110 \le E_1 \le 130 \ GPa$$

 $20 \le E_2 = E_3 \le 35 \ GPa$
 $5 \le G_{12} = G_{13} \le 7 \ GPa$

Then, in the points of the plan of the experiments, finite element analysis was carried out to determine the first eleven frequencies.

Employing these numerical values, the approximating functions (response surfaces) for all eigenfrequencies were obtained with the correlation coefficients c=98 %. The program RESINT was used to obtain the equation of regressions. As an example, these approximations with correlation coefficients are given below for the first eigenfrequency:

 $f_1 = 166.7 + 4.78 \cdot z_1 + 3.99 \cdot z_2 + 1.68 \cdot z_3,$

where the normalized variables z_i are introduced:

$$z_1 = -12.0 + 0.10 \cdot 10^{-9} \cdot x_1$$

$$z_2 = -3.67 + 0.13 \cdot 10^{-9} \cdot x_2$$

$$z_3 = -6.00 + 0.10 \cdot 10^{-8} \cdot x_3$$

The elastic constants of the laminated composite panel with one stringer were obtained by minimizing the error functional (7). These values are presented in table 1.

Table 1. Elastic constant obtained by identification

Property	Panel	
E_1 , GPa	120.80	
$E_2 = E_3$, GPa	24.80	
$G_{12} = G_{13}$, GPa	5.80	

After the finite element calculations at the reference points of the plan of the experiment, the mode shapes of experimentally measured and numerically calculated eigenfrequencies were compared. Some typical vibration modes of both the experimentally measured and the respective numerically calculated eigenfrequencies of the stiffened composite panel are presented in figures 9 and 10 (a-experimentally measured; b-numerically calculated).



The experimentally measured frequencies presented in table 2 can be used for identification in any combination. A cross validation for all sample points was performed to achieve a better approximation of the original function and to select the most important (most sensitive to elastic constants) and reliable frequencies. The elastic constants obtained by identification were verified by comparing the experimentally measured eigenfrequencies with the numerical ones obtained by FEM at the point of optima (using the elastic properties identified).

Fable 2. Frequencies	s and	residuals	for	panel	
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Mode	$f_i^{\exp}(\text{Hz})$	f_i^{FEM} (Hz)	Δ (%)
1	166.4 ^a	165.4	0.6
2	236.8 ^a	237.2	0.2
3	261.2 ^a	261.1	0.0
4	289.5 ^a	289.8	0.1
5	301.3 ^a	299.7	0.5
6	411.5 ^a	412.1	0.1
7	431.3 ^a	432.6	0.3
8	-	553.6	
9	570.2	583.0	2.2
10	741.0	722.5	2.5
11	759.3	761.6	0.3
9			

^a the frequencies used in identification

The residuals Δ_i are calculated by the expression (Tab. 2):

$$\Delta_i = \frac{\left| f_i^{FEM} - f_i^{exp} \right|}{f_i^{exp}} \times 100 \tag{8}$$

It is seen from the comparison of the results presented in table 2 that frequencies calculated by the finite element method using the elastic properties obtained by identifycation are in good agreement with the experiment.

5. Conclusions

The identification of the actual material properties was performed on a small-stiffened panel cut out from a large three-stringer panel. The results obtained are sufficiently accurate for the in-plane longitudinal Young's modulus and for the in plane shear modulus. The transverse Young's modulus differs significantly from the nominal constants of carbon-fibre-reinforced composites. These discrepancies are explained by the fact that the parameters of the real structure differ from the nominal values (layer thickness, layer angles, the material density is not so homogeneous in all parts of the stiffened panel, etc.) of laminated composites. These differences should be taken into account in designing the real structures by choosing the safety factors and calculating the limit and collapse loads of composite stiffened structures.

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STANDAUS KOMPOZITINIO KEVALO ELASTINIŲ SAVYBIŲ IDENTIFIKAVIMAS

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Santrauka

Aprašomas skaitinis-eksperimentinis briaunotos kompozitinės panelės tamprumo savybių nustatymo metodas. Šioms savybėms nustatyti taikytas identifikavimo metodas, kuriame sujungti skaitinio eksperimento planavimas, skaičiavimas baigtinių elementų metodu bei gautų duomenų aproksimavimas. Identifikavimo rezultatus patvirtina pačios panelės dažnių eksperimentinių duomenų palyginimas su skaitiniais baigtinių elementų metodu aptiktų tamprumo savybių rezultatais.

Reikšminiai žodžiai: baigtinių elementų modelis, identifikavimas, valdomas paviršius, vibracijos testas, POLYTEC.