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EVOLUTION OF CORROSION OF CIVIL AIRCRAFT BASED ON IMPROVED GREY MODELS

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Abstract. Structure corrosion is one of the most common damages affecting the structural integrity of the aging civil aircraft. Three grey models were applied for predicting the corrosion evolution during aircraft maintenance checks. The developed models include the basic GM (1, 1) model and two improved models with the initial condition optimized by linear transformation and partial differential methods, respectively. Both improved models show better quantitative agreement with the existing data, while the model using the partial differential method exhibits the highest prediction accuracy amongst the three models presented above. Such models can also be used on the structure of other complex equipment to improve the efficiency of preventive maintenance.

Keywords: corrosion, civil aircraft, grey models, evolution, prediction.

Introduction

Nowadays, commercial aircraft fleets around the world are flying much longer than anticipated when their design service objectives (DSO) were usually 20 calendar years (He, Li, & Zhang, 2016). Due to the trade-off between operation and procurement costs, the service life of commercial aircraft is usually 1.5~2 times of their DSO. Great concerns about the structural integrity of these aging aircraft are being addressed from many perspectives (Hoeppner & Arriscorreta, 2012). A major cause for concern is corrosion, accompanied with widespread fatigue damage (Jones, 2014).

The prediction of corrosion evolution plays a key role in the evaluation of structural damage and subsequent prediction of the residual strength and fatigue life. Some research has been done to develop corrosion prediction models (Altay, Ozkan, & Kayakutlu, 2014; Shekhter, Crawford, & Loader, 2015; Yang, Zhang, Guo, & Wang, 2016). However, most of the previous studies mainly focus on statistical models, damage mechanics, or neural networks to predict aviation aluminum alloy corrosion behavior. It has been well known that causes for the corrosion of civil aircraft are very complex, such as operation environments, design levels, maintenance intervals and so on. It is also very difficult to quantitatively evaluate the extent or nature of corrosion damage.

However, a new method, the grey system theory is suitable for studying such an uncertain problem with less data and poor information. The application scope of grey system theory has extended to industry, social affairs, economy, energy, financial and other fields (Xu, 2015; Liu, Tao, & Xie, 2016; Zeng, Luo, & Liu, 2016; Rathnayaka, Seneviratna, & Wei, 2015). Nevertheless, there are few efforts dealing with the corrosion evolution behavior in civil aircraft fleets. Consequently, the major objective of this work is to characterize the corrosion evolution behavior of civil aircraft using improved grey system models. Prediction results are obtained to illustrate the trends resulting from various influence factors.

1. Constructing the GM (1, 1) model

The GM (1, 1) is the most widely used model of the grey prediction theory (Deng, 1982; S. Liu, Zeng, J. Liu, Xie, & Yang, 2015). It denotes a single variable first-order linear model which can be applied by using a limited number of data observations. It deals with uncertain systems with partially known information by obtaining useful information from what is available.

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The following steps are carried out to obtain the GM (1, 1) model. Firstly, the raw data sequence can be denoted by:

$$X^{(0)} = \{x^{(0)}_{(1)}, x^{(0)}_{(2)}, \ldots, x^{(0)}_{(n)}\} . \quad (1)$$

Then, the accumulated generation operation (AGO) is applied to obtain the sequence X(1):

$$X^{(1)} = \{x^{(1)}_{(1)}, x^{(1)}_{(2)}, \ldots, x^{(1)}_{(n)}\} , \quad (2)$$

where

$$x^{(1)}_{(k)} = \sum_{i=1}^{k} x^{(0)}_{(k)}, k = 1, 2, \ldots, n . \quad (3)$$

Secondly, using the mean consecutive neighbors operator for X(1), then the sequence Z(1) is obtained:

$$Z^{(1)} = \{z^{(1)}_{(1)}, z^{(1)}_{(2)}, \ldots, z^{(1)}_{(n)}\} , \quad (4)$$

where

$$z^{(1)}_{(k)} = 0.5x^{(1)}_{(k)} + 0.5x^{(1)}_{(k-1)}, k = 2, 3, \ldots n . \quad (5)$$

Then, a differential equation is constructed in the accumulated data sequence with respect to time. The GM (1, 1) model can be built through a first-order differential equation:

$$\frac{\mathrm{d}x^{(1)}_{(t)}}{\mathrm{d}t} + ax^{(1)}_{(t)} = u . \quad (6)$$

Parameter -*a* reflects the improvement in the trend of the sequence, thus it is called the developmental coefficient. Parameter *u* reflects the change in relationships and is called the coordination parameter.

Based on Eq. (6), the original form of GM (1, 1) model for discrete values based on *z* can be defined in Eq. (7):

$$x^{(0)}_{(k)} + az^{(1)}_{(k)} = u . \quad (7)$$

In Eqs (7), *a* and *u* are the parameters to be estimated as follows by minimizing the squared-errors, i.e., least squares estimation:

$$\begin{pmatrix} \hat{a} \\ \hat{u} \end{pmatrix} = \left[B^T B \right]^{-1} B^T Y , \quad (8)$$

where

$$B = \begin{bmatrix} -(x^{(1)}_{(1)} + x^{(1)}_{(2)})/2 & 1 \\ \cdots & 1 \\ -(x^{(1)}_{(n-1)} + x^{(1)}_{(n)})/2 & 1 \end{bmatrix} , \quad (9)$$

$$Y = \left[x^{(0)}_{(2)}, x^{(0)}_{(3)}, \ldots x^{(0)}_{(n)} \right]^T . \quad (10)$$

The matrix *B* and *Y* imply the accumulated matrix and constant vector, respectively.

Based on the parameters, the time response function can be obtained by solving Eq. (6), as shown below:

$$\mathrm{x}^{(1)}_{(k+1)} = \left[x^{(1)}_{(1)} - \frac{\hat{u}}{\hat{a}} \right] e^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}}, k = 1, 2, \ldots, n-1 . \quad (11)$$

The initial condition of the model is $x^{(1)}_{(1)} = x^{(0)}_{(1)}$.

The predicted series for the original data sequence can be calculated by inverse accumulated generation operation, as given in the following equation:

$$\mathrm{x}^{(0)}_{(k+1)} = x^{(1)}_{(k+1)} - x^{(1)}_{(k)}, k = 1, 2, \ldots n-1 . \quad (12)$$

2. Improving the GM (1, 1) model

From the above-mentioned steps of the GM (1, 1) model, it can be seen that the initial value of X(1) is simply set to be equal to the first raw data, i.e. x(1)(1) = x(0)(1). In other words, the prediction curve of the GM (1, 1) model is just set to the first raw data. In many cases, however, this method would lead to great prediction errors for the subsequent data (Dang, S. Liu, & B. Liu, 2005). Similarly, the initial value of X(1) is simply set to be equal to the last raw data, or anyone of the raw data, which would result in the same situation. Therefore, the initial condition of X(1) should be optimized for best consistency with the real development trend. For this purpose, two optimization methods of the initial condition of X(1) are studied and applied to the prediction of civil aircraft corrosion.

2.1. Optimization by linear transformation

For similarity, assume the AGO data X(1) is denoted by

$$X^{(1)} = ce^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}} . \quad (13)$$

After linear transformation, the following equation is obtained

$$\ln(X^{(1)} - \frac{\hat{u}}{\hat{a}}) = -\hat{a}k + \ln c . \quad (14)$$

Let Y = $\ln(X^{(1)} - \frac{\hat{u}}{\hat{a}})$, A = $-\hat{a}$, C = ln *c*, thus the equation can be transformed to

$$Y = Ak + C . \quad (15)$$

For this linear equation, given the real sequential data, the constant of *C* can be optimized by LSM as:

$$\hat{C} = \frac{\sum_{k=1}^{n} Y - A\sum_{k=1}^{n} k}{n} . \quad (16)$$

Thus, the coefficient of *c* of the equation can be obtained by:

$$\hat{c} = \begin{cases} \exp\left[(\sum_{k=1}^{n} \ln(x^{(1)}_{(k)} - \frac{\hat{u}}{\hat{a}}) + a\sum_{k=1}^{n} k)/n \right], & \text{if}(x^{(1)}_{(k)} - \frac{\hat{u}}{\hat{a}}) > 0 \\ \exp\left[(\sum_{k=1}^{n} \ln(\frac{\hat{u}}{\hat{a}} - x^{(1)}_{(k)}) + a\sum_{k=1}^{n} k)/n \right], & \text{if}(x^{(1)}_{(k)} - \frac{\hat{u}}{\hat{a}}) < 0. \end{cases} \quad (17)$$

Since the parameters have all been obtained, the equation can be used to predict the trend of civil aircraft corrosion.

2.2. Optimization by partial differential

For the equation, the optimization goal is usually to get the least square, namely, to find the best combination of the parameters to minimize the following function:

$$Z = \sum_{k=1}^{n} [X^{(1)} - (ce^{-ak} + \frac{u}{a})]^2 . \quad (18)$$

Thus, the best combination of the parameters should be obtained by solving the following partial differential equation group:

$$\begin{cases} \frac{\partial Z}{\partial c} = c\sum e^{-2ak} + \frac{u}{a}\sum e^{-ak} - \sum x^{(1)}{}_{(k)} e^{-ak} = 0 \\ \frac{\partial Z}{\partial a} = c\sum ke^{-2ak} + \frac{u}{a}\sum ke^{-ak} - \sum kx^{(1)}{}_{(k)} e^{-ak} = 0 \\ \frac{\partial Z}{\partial u} = c\sum e^{-ak} + n\frac{u}{a} - \sum x^{(1)}{}_{(k)} = 0 \end{cases} . \quad (19)$$

Unfortunately, this group is of transcendental equations with high orders. So far, there is no available method to solve it. Since the parameters a and u can be obtained by the GM (1, 1) model, the coefficient of c can be solved as:

$$\hat{c} = \frac{\sum_{k=1}^{n} e^{-\hat{a}k} (x^{(1)}{}_{(k)} - \frac{\hat{u}}{\hat{a}})}{\sum_{k=1}^{n} e^{-2\hat{a}k}} . \quad (20)$$

Thus, the equation can also be used to predict the trend of civil aircraft corrosion.

3. Application of improved grey models

During the operation of civil aircraft, the amount of corrosion that occurred is random, and is affected by many factors, such as environments, coating qualities, design levels, maintenance intervals, and so on. The scheduled maintenance tasks identifying the frequency of an accomplishment in the airline maintenance base are often in terms of letter checks; e.g., 1C, 2C, 3C, etc. The C check interval is about 18 months and, at 8C check, the aircraft will be subjected to an overhaul during which severely corroded parts will be removed and replaced after a long term service, and various corrosion inhibitor processing required by the Corrosion Prevention Manual (CPM) will be carried out. Thus, in theory, the corrosion damage will be controlled under the same level as for a new aircraft. A typical sequential number of corrosion detected for a civil aircraft in terms of the C check up to the 8C check is listed in Table 1. It was derived from the raw corrosion-related task data, which was acquired from a maintenance base of an airline in China. The statistical result indicates that the frequently corroded zone is 100 (lower fuselage), 200 (upper fuselage), and 400 (power plant and nacelle struts) (according to ATA100 specification), while the most frequently corroded parts are the skin, floor beam and frame. The corroded depth is usually less than the allowed damage depth defined by the Structure Repair Manual (SRM), which can be classified as level 1 corrosion according to the Corrosion Prevention and Control Program (CPCP). Level 2 corrosion was found very occasionally during the maintenance checks, which demonstrates the validity of the CPCP implemented in the currently studied airline.

According to the GM (1, 1) model, the developmental coefficient and coordination parameters can be calculated as $-a = 0.3516$, $u = 3.6581$.

Thus, the accumulated sequence X(1) can be described as:

$$X^{(1)}{}_{(k+1)} = 15.4031e^{0.3516k} - 10.4031, k = 1, 2, \ldots, n-1 . \quad (21)$$

Then, the estimated number of corrosion can be calculated by Eq. (21) and is listed in Table 1.

Similarly, the initial condition can be optimized through the methods of linear transformation and partial differential, respectively. Consequently, the accumulated sequence X(1) can be described as Eq. (22) and Eq. (23), respectively.

$$X^{(1)}{}_{(k+1)} = 11.2345e^{0.3516k} - 10.4031, k = 1, 2, \ldots, n-1 . \quad (22)$$

$$X^{(1)}{}_{(k+1)} = 11.3284e^{0.3516k} - 10.4031, k = 1, 2, \ldots, n-1 . \quad (23)$$

The estimated number of corrosion can also be calculated and is listed in Table 1 for comparison.

The relative error is usually described as:

$$\varepsilon_{(k)} = \left| \frac{\hat{x}^{(0)}_{(k)} - x^{(0)}{}_{(k)}}{x^{(0)}{}_{(k)}} \right| . \quad (24)$$

The relative errors of each method at every maintenance interval are also shown in Table 1 for comparison.

Table 1. Evolution of corrosion detected at maintenance multiple intervals

Maintenance intervals	Number of corrosion	Predicated by BGM[a]	Relative error	Predicated by ILT[b]	Relative error	Predicated by IPD[c]	Relative error
1C	5	5	0%	5	0	5	0
2C	7	6.49	7.27%	6.72	3.86%	6.78	3.06%
3C	10	9.23	7.73%	9.56	4.35%	9.64	3.55%
4C	14	13.11	6.32%	13.59	2.89%	13.71	2.07%
5C	20	18.64	6.79%	19.32	3.37%	19.49	2.56%
6C	26	26.49	1.90%	27.47	5.65%	27.69	6.52%
7C	40	37.66	5.84%	39.04	2.39%	39.37	1.57%
8C	57	53.53	6.08%	55.49	2.63%	55.96	1.83%

Note: a – BGM denotes the GM (1, 1) model; b – ILT denotes the model improved by linear transformation; c – IPD denotes the model improved by partial differential.

The mean relative error is described as:

$$\bar{\varepsilon} = \sum_{k=1}^{n} \left| \frac{\hat{x}^{(0)}_{(k)} - x^{(0)}{}_{(k)}}{x^{(0)}{}_{(k)}} \right|. \quad (25)$$

Moreover, the measurement of the grey relational degree between the raw data and the predicted values can be used to indicate the degree of prediction accuracy. The most common grey relational degree can be described as:

$$r(x_{\text{raw}}, x_{\text{pred}}) = \frac{1}{n}\sum_{k=1}^{n} r(x_{\text{raw}}(k), x_{\text{pred}}(k)), \quad (26)$$

where

$$r(x_{\text{raw}}(\text{k}), \text{x}_{\text{pred}}(\text{k})) = \frac{\min_i \min_k \left|x_{\text{raw}}(\text{k})\text{-x}_{\text{pred}}(\text{k})\right| + \rho \max_i \max_k \left|x_{\text{raw}}(\text{k})\text{-x}_{\text{pred}}(\text{k})\right|}{\left|x_{\text{raw}}(\text{k})\text{-x}_{\text{pred}}(\text{k})\right| + \rho \max_i \max_k \left|x_{\text{raw}}(\text{k})\text{-x}_{\text{pred}}(\text{k})\right|}. \quad (27)$$

where ρ is referred to as the identification coefficient and is usually set to 0.5.

The mean relative error, square error, and grey relational degree of the three methods are shown in Table 2. It can be seen that the mean relative error of the GM (1, 1) model is the largest (5.25%), while the mean relative error of the improved model with linear transformation has decreased to 3.15%, and the partial differential method has the least mean relative error (2.64%). This result is consistent with the sequence of prediction accuracy in terms of square error. Since the goal of the model with the partial differential method is to minimize the square of errors, there is no doubt that the least square error is obtained by this model.

According to the nature of grey relational degree, a greater grey relational degree of the models indicates better prediction accuracy. From Table 2, it can be seen that the grey relational degree of the improved model with the partial differential method is the largest (0.776), while the grey relational degree of the improved model with the linear transformation has decreased to 0.739 and, for the GM (1, 1) model, it has the least relational degree (0.653). This is in accordance with the above-mentioned prediction accuracy sorted by the relative error. Therefore, all the comparisons prove that the proposed improved model with the partial differential method has higher prediction ability in terms of relative error, square error and grey relational degree.

Table 2. Comparison of the prediction accuracy of the proposed grey models

Prediction method	Mean relative error	Square error	Grey relational degree
BGM[a]	5.25%	21.25	0.653
ILT[b]	3.15%	6.22	0.739
IPD[c]	2.64%	4.88	0.776

Note: a – BGM denotes the GM (1, 1) model; b – ILT denotes the model improved by linear transformation; c – IPD denotes the model improved by partial differential.

Conclusions

The corrosion evolution of a civil aircraft structure can be predicted by improved grey models with an initial condition optimized by linear transformation and partial differential methods. Both improved grey models show higher prediction accuracy than the GM (1, 1) model. The effective estimation of the number of corrosion of civil aircraft structures could promote the efficiency of aircraft structure maintenance of the airline industry. The proposed models could also be applied for the corrosion estimation of other large, complex equipment for preventive maintenance. However, due to the single-variable nature of the grey model studied, it cannot be used to simultaneously predict the evolution of other parameters such as corroded depth, level, position or even exact part. This could be studied further by combining artificial neural networks with grey models to acquire multiple output variables.

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