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MATHEMATICAL MODELS OF THE CALCULATION OF AIRCRAFT STRUCTURAL RELIABILITY

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Abstract. The mathematical models of damageability under static and cyclic loadings are investigated. The parameters of damageability and endurance are considered necessary for the calculation of long-term durability wear and low-cycle and high-cycle fatigue, thermo-cyclic endurance and deformation as well as general models, taking into account multi-axis and multicomponent loads. The models of damageability at loading range, the probabilistic models of endurance and accumulation of damage at random influence are given. Probabilistic criteria for service life estimation and their connection with accumulated damageability are researched.

Keywords: long-term durability, low-cycle fatigue, multicomponent loading, high-cycle fatigue, damageability models, complex stressing condition, spectrum of stresses, crack resistance.

1. Introduction

For the constructive elements with long periods of latent damage accumulation, it is expedient to evaluate operational condition on the basis of individual service life calculation depending on stressing during the operation period.

In the process searching for rational constructive and engineering solutions, which allow an increase in the reliability of a specific component, is always the result of the analysis and the compromise between features of various aspects of stressing, a calculation of probability for possible types of destruction, its consequences, an engineering and economic ground of the appointed service life. Such an approach to service life calculation can be implemented by means of the rated-experimental methods providing comparison between parameters of operational loading of components with the experimental performances of their durability considering the influence of the main factors of actual loading processes.

2. Models of component damaging during static and cyclic loading

During the calculation of the service life of crucial constructive elements of airframe and gas turbine engines (GTE) at the design stage and while in service, the concept of material damaging is applied.

Damaging is understood as the process of irreversible changes developing in a material under the influence of stress, strain and temperature, and leading, finally, to destruction.

As physical phenomenon this process is the irreversible change of material structure, which causes the violation of material integrity (macro cracks, form change, warping, etc.), defined by the character of static, long static, multi-cyclic, low-cycle, and thermo-cyclical operating loading.

Material damageability is estimated by the parameters that describe behaviour of a material on the basis of mechanical methods of a continuous solid state.

The material damageability rate of a component is estimated by relative value D, which varies within the limit 0÷1. Its value on an initial intact condition equals zero ($D = 0$), and at the moment of reaching a limiting condition equals one ($D = 1$).

For the evaluation of long static damage under the condition of mono axial loading the conditional principle of linear damage summation in the form of relative endurance is used.

$$D_s = \int_0^t \frac{dt}{\tau_{dur}(\sigma_{st}, T)},$$

where τ_{dur} – characteristics of long durability.

Multi-axis loading is considered to be one of four equivalent loadings which are a combination of the principal loadings σ_1, σ_2 и σ_3 ($\sigma_1 \geq \sigma_2 \geq \sigma_3$):

$$\sigma_{e1} = \sigma_1$$

$$\sigma_{e2} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

$$\sigma_{e3} = \frac{1}{2}(\sigma_{e1}+\sigma_{e2})$$

$$\sigma_{e4} = (\sigma_1+\sigma_3).$$

We use following equations for the description of performance for long durability $\tau_{dur}(\cdot)$ at fixed temperature or exponential dependences of time before destruction t^* from equivalent stress σ_e :

$$t^*(\sigma_e) = C\sigma_e^{-n}$$

$$t^*(\sigma_e) = A\exp(-\alpha\sigma_e)$$

where C, n, A, α – material constants.

At variable temperature the generalised performance $\lg(t^*) = \tau(\sigma, T)$ of long durability is used. Three types of these equations that describe dependences with accuracy for alloy ЖС26ВСНК (Fig. 1) are given in (Кулик *и др.* 2008).

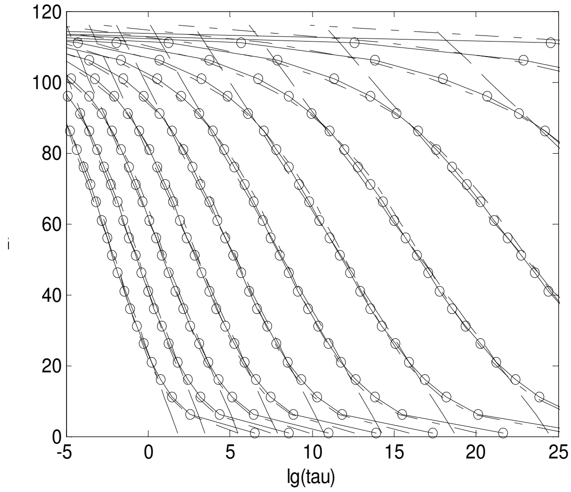


Fig. 1. Types of long-term durability curves of alloy MAR (ЖС26ВСНК):

$$\lg(t^*) = -20 + (36537.33 - 9082.95\lg(\sigma) + 4180.43\ \lg(\sigma)^2 - 1569.85\ \lg(\sigma)^3 + 341.732/(\lg(\sigma) - 2.08))/T)$$

$$\lg(t^*) = -20 + (35273.76 - 21806.17\lg(\sigma) + 2383.192\ \lg(\sigma)^2)/(1 - 0.466814\ \lg(\sigma))/T$$

$$\lg(t^*) = -20 + 30004.82/T - 106.8396\sigma/T$$

where σ – component loading in kg/mm^2; T – component temperature °K.

The method of estimating fatigue ratio is the same:

$$D_c = \int_1^N \frac{dN}{N(\sigma_a, T)},$$

where $N(\cdot)$ – endurance performances.

In case of multi component loading for calculation of component damageability, such criteria of destruction as strength, time and deformation are applied (Трощенко *и др.* 2009; Расчет... 1984). These criteria are based on both linear and non-linear methods of damage calculation.

3. Damage models of components by loading range

The components of glider and power plant are usually under the influence of changing stress with different maximum stress values $\sigma_{\max}$. A loading range is a stress frequency of varying intensity. Changing loading with different maximum stress values $\sigma_{\max}$ usually acts on the airframe and engine components during the flight. Let us analyse a range of stresses occurring from the bending momentum that affects wing components caused by atmospheric turbulence. The work reveals the analysis of the flight-stressing ratio of different aircraft. It shows that the range of its amplitudes complies with the logarithmic law $\sigma_a = f(\ln(N_\sigma))$ and lies within the same range with changes (Schijve *et al.* 1973). The loading range can be stated as the ratio σ_f / σ_c, where σ_f is the current stress value and σ_c is the average stress value in flight, and it characterises the flight conditions. The continuous loading range for limiting the stress can be replaced by a step function that greatly reduces the complexity The range is divided into 10 degrees with a maximum value σ_1 / σ_c =1.6 and minimum σ_{10} / σ_c =0.222 (Tab. 1).

Table 1. Discrete stresses range

σ_i / σ_c	1,6	1,50	1,30	1,15	0,995
n_σ	1	2	5	18	52
σ_i / σ_c	0,84	0,685	0,530	0,375	0,2220
n_σ	152	800	4170	34800	358665

Maximum loading of the final two stages and corresponding stress intensity factors (SIF) for crack resistance analysis are lower than the level of the saturation intensity threshold coefficient K_{th}. That is why they are not taken into account. The lower part of table 1 shows correspondence to loading cycles n_σ of a given level, which is within the prescribed limits. In this block, maximum stress appears once, while minimum stress appears 4170 time per every 5200 loading cycles.

It's obvious that

$$\sigma_{\max} = \sigma_c + \sigma_a ;$$
$$\sigma_{\min} = \sigma_c - \sigma_a .$$

Cycle asymmetry factor:

$$r = \frac{\sigma_{\min}}{\sigma_{\max}} .$$

Figure 2 shows maximum and minimum stresses of range loadings by given the mid-value of the loading σ_c = 70 MPa (Кулик *и др*. 2009).

The equations of the curves for maximum and minimum stresses range by $\sigma_c = 70$ MPa look like:

$\lg(N_{\max i})$=0.0003477 $\sigma^2_{\max i}$ -0.14892 $\sigma_{\max i}$ +15.669;

$\lg(N_{\min i})$=0.0003477 $\sigma^2_{\min i}$ +0.05155 $\sigma_{\min i}$ +1.6357;

where

$$\sigma_{max\ i} = \sigma_c (1 + \frac{\sigma_i}{\sigma_c}); \ \ \sigma_{min\ i} = \sigma_c (1 - \frac{\sigma_i}{\sigma_c})$$

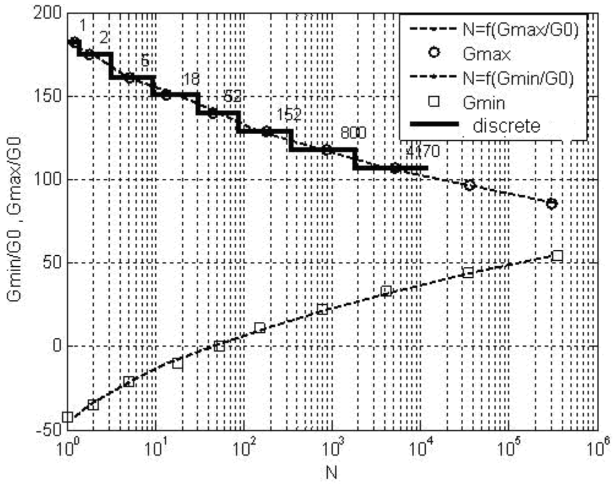


Fig. 2. Continuous and discrete loading range

Load program is based on the formation of particular ranges that in their turn are formed by the main range by exclusion of its particular components: the first one, first two ones, etc. Meanwhile, each range characterises stressing conditions during a particular flight. When there is a full range similar flight conditions occur in a turbulent atmosphere (storm), in other cases in less dangerous situations. Occasional order of the loading range choice and programming allows us to simulate flight conditions close to real ones.

According to the linear damage summation hypothesis, the extent of damage is proportional to the relation of cycle number $n_{\sigma i}$ to the limit number of cycles $N_{\lim i}(\sigma_{\max i})$ for a given i^{th} stress $\sigma_{\max i}$. The discrete loading range has k levels, and the continuous one has loading cycles, a part of which depends on a stress level that changes from $\sigma_{\max 1}$ to $\sigma_{\max k}$.

Let us call the general number of loading cycles 'a full cycle', the obtained damage 'a damage range', and total accumulated damage 'damage per one full stress cycle'. Due to this, the formulae for calculation of accumulated damage per *m* full loading cycles in the discrete and continuous variant is following:

$$D_m = \sum_{l=1}^{m} \sum_{i=1}^{k} \frac{n_{\sigma i}(\sigma_{\max i})}{N_{\lim i}(\sigma_{\max i})} , \quad (1)$$

$$D_m = \sum_{l=1}^{m} \int_{\sigma_{\max 8}}^{\sigma_{\max 1}} \frac{n_\sigma(\sigma_{\max}) d\sigma}{N_{\lim}(\sigma_{\max})} , \quad (2)$$

where damage ratio is $D_m > 1$.

In a general the quantity $n_{\sigma i}$ defines only that part of cycles that corresponds to a given stress level $\sigma_{\max i}$; that is why it can be a non-integer. In a given example of the loading range the maximum stress appears only 10 times per 40 000 flights (Schijve *et al.* 1973); that is why while calculating the damage for one flight $n_{\sigma i}$ =1/4000.

Taking into account the complicated nature of actual component loading ranges and integration kind of resistance properties when computing the value of integrals, it is appropriate to use quadrature formulae – the Simpson one and the Newton-Cotes one of the 8th degree. But for some cases there are end-design formulae of the damage model per full loading cycle of the component:

The first model along with linear range dependence and limit stresses from $\lg(N)$:

$$D_{FC} = \int_{v_1}^{v_2} \frac{n_\sigma(\sigma)d\sigma}{N(\sigma)} = \int_{v_1}^{v_2} \frac{10^{c+d\sigma}}{10^{a+b\sigma}} d\sigma = \\ = \frac{10^{(c-b_{0+v_2}(d-b_1))} - 10^{(c-b_{0+v_1}(d-b_1))}}{(d-b_1)ln(10)} .$$

The second model along with quadratic range dependence and the linear one of limit loading from lg*(N) is given below:*

$$D_{FC} = \int_{v_1}^{v_2} \frac{n\ (\sigma)d\sigma}{N(\sigma)} = \int_{v_1}^{v_2} \frac{10^{c+d\sigma+q\sigma^2}}{10^{a+b\sigma}} d\sigma =$$

$$= \left(\begin{array}{l} -\frac{1}{2} erf\left(\frac{1}{2} \frac{q v_2 ln(100) + ln(10)(d-b_1)}{\sqrt{-q ln(10)}} \right) + \\ \frac{1}{2} erf\left(\frac{\frac{1}{2} q v_1 ln(100) + \frac{1}{2} ln(10)(d-b_1)}{\sqrt{-q ln(10)}} \right) \end{array} \right) \times$$

$$\times \exp\left(\frac{ln(10)(q(c-b_0) - \frac{1}{4}(-d(d-2b_1) - b_1^2)}{q} \right) \Big/ \sqrt{-\frac{q ln(10)}{\pi}} .$$

The third model along with range degree dependences and limit loads:

$$D_{FC} = \int_{v_1}^{v_2} \frac{n_\sigma(\sigma)d\sigma}{N(\sigma)} = \int_{v_1}^{v_2} \frac{A\sigma^m}{B\sigma^n} d\sigma = \\ = \frac{A\left(-v_2^{-n+m+1} + v_1^{-n+m+1}\right)}{B(n-m-1)} .$$

The fourth model along with strength-exponential range dependence and limit load

$$D_{FC} = \int_{v_1}^{v_2} \frac{n_\sigma(\sigma)d\sigma}{N(\sigma)} = \int_{v_1}^{v_2} \frac{A\sigma^m c^\sigma}{B\sigma^n b_1^\sigma} d\sigma =$$

$$\frac{-A\left(\begin{array}{l} C_{v1}^{22} n - C_{v1}^{22} m + C_{v2}^{11} n - C_{v2}^{11} m - \\ -C_{v1}^{21} n + C_{v1}^{21} m - C_{v2}^{12} n + C_{v2}^{12} m + \\ +C_{v2} - C_{v1} \end{array} \right)}{((-ln(c) + ln(b_1))B)} ,$$

where $C_{v1}^{22} = v_1^{-n+m} d_1^{n-m} \gamma(-n+m, d_1)$;

$$C_{v2}^{11} = v_2^{-n+m} d_2^{n-m} \Gamma(-n+m);$$

$$C_{v1}^{21} = v_1^{-n+m} d_1^{n-m} \Gamma(-n+m);$$

$$C_{v2}^{12} = v_2^{-n+m} d_2^{n-m} \gamma(-n+m, d_2);$$

$$C_{v2} = v_2^{-n+m} c^{v2} b_1^{-v2};$$

$$C_{v1} = v_1^{-n+m} c^{v1} b_1^{-v1};$$

$$d_1 = v_1(-ln(c) + ln(b_1));$$

$$d_2 = v_2(-ln(c) + ln(b_1));$$

$\Gamma(\alpha)$, $\gamma(\alpha, x)$ – full and partial range of functions.

The continuous line of figure 3 shows the stress range (SR) and stress limit (LS), and the dotted line shows damage range per one full stress cycle for 1 model:

SR: $\lg(N_S)$ = -0.554125 σ_{max} +9.7235455;

LS: $\lg(N_{D1})$= 12.294422-0.275464 σ_{max} .

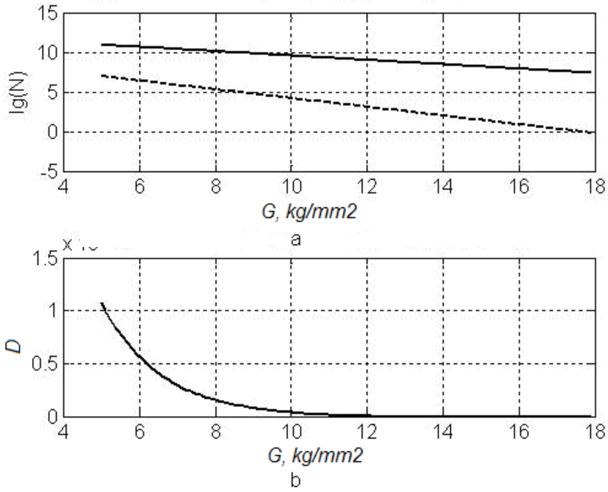


Fig. 3. The change of first model per full cycle: a – stress range and durability characteristics $\lg(N) = f(\sigma) + S_\sigma$ b – damage range of a full cycle $D = f(\sigma) + S_\sigma$

Figure 4 shows the same characteristics for quadratic variation $\lg(N_S)$=0.03477 σ_{max}^2 -1.4892 σ_{max} +15.669) and stress limit $\lg(N_{D2})$=23.4151-5.6135 ln(σ_{max}), calculated with the help of the numerical method. The quantities of accumulated damage per one full cycle equal: first model - D_1 = 0,0014, second model - D_2 =0.0001151.

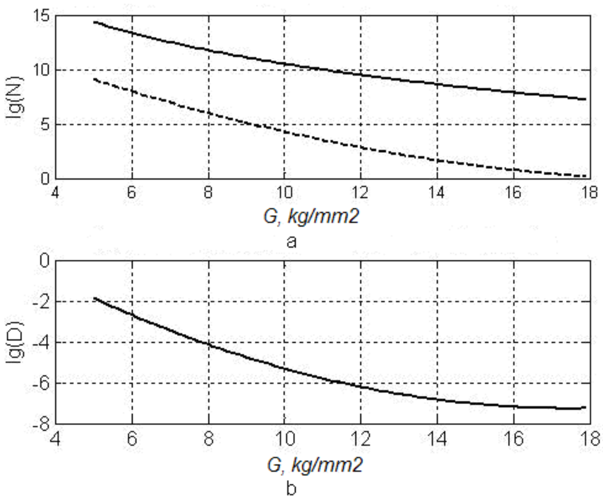


Fig. 4. Characteristics of the second model per full cycle: a – loading range and durability characteristics $\lg(N) = f(\sigma) + S_\sigma$ b – damage range of a full cycle $D = f(\sigma) + S_\sigma$

4. Probabilistic damage models under static and repeated loading

Let us consider several possible component damage accumulation process models during the flight.

The random lumped loadings slowly vary in accordance with the function that acts on components (Fig. 5):

$$\sigma(t)=\sigma_0+\chi(t), \tag{6}$$

where σ_0 – random variety, distributed in accordance with ordinary law with following parameters $M[\sigma_0]=0$, $D[\sigma_0]=S^2_0$;

$\chi(t)$ - determinate function.

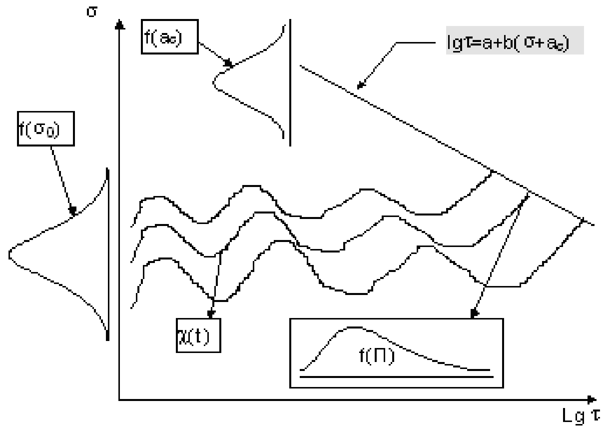


Fig. 5. The damage progress model under random stress

The influence on the component in flight stress rates during the entire flight will differ by the constant value from the average values for these modes For example, this may be due to changing weather conditions, environment and other external factors of the flight.

The structural behaviour of component material under the influence of static or slowly varying loads and constant temperature can be described by a stress rupture curve, which is valid for the exponential law of relationship between destructive stress σ_n and the time to failure.

In probabilistic interpretation this curve has the following structure:

$$\lg\tau = a + b(\sigma + a_\sigma), \tag{7}$$

where a_σ – is probabilistic material properties variable;

$$M[a_\sigma]=0,\ D[a_\sigma]=S^2_a.$$

According to the linear hypothesis of static damage summation, the level of damage accumulated in the material components of damage D during the time of loading t^* is defined by the formula:

$$D\left(t^*\right)=\int_0^{t^*}\frac{dt}{\tau\left(\sigma(t)\right)}.$$

Substituting expressions (6) and (7) in this formula, we obtain

$$D=\int_0^{t^*}\frac{dt}{exp\left(a+b\left(\sigma(t)+a_\sigma\right)\right)}=\frac{\gamma\left(t^*\right)}{exp\left(a+b\left(\sigma_0+a_\sigma\right)\right)}=\varphi\left(\sigma_0+a_\sigma\right) \tag{8}$$

where

$$\gamma\left(t^*\right)=\int_0^{t^*}\frac{dt}{\exp\left(b\chi(t)\right)}$$

$\gamma(t^*)$ – determinate function.

The rate of the random variety of damageability per flight D in expression (8) is connected with the random variety σ_0 and steady exponential dependence a_σ. To determine the distribution damage density $f(n)$ we use the formula

$$f_D(D) = f_\nu(\Psi(D))|\Psi'(D)|, \tag{9}$$

where $f_\nu(\cdot)$ – distribution damage density of the sum of normal independent random variable σ_0 and a_σ; $M[\nu]=0$; $D[\nu]=S^2_\nu=S^2_a+S^2_\sigma$;

$\Psi(D)$ - inverse function in relation to $\varphi(\cdot)$.

According to the expression (9)

$$\Psi(D) = \nu = (\gamma(t^*) - \ln D - a)/b;$$

$$|\Psi'(D)| = 1/bD.$$

Returning to formula (9), we have

$$f_D(D)=\frac{1}{\sqrt{2\pi}S_0bD}\exp\left(-\frac{\left(\ln D-\ln\gamma\left(t^*\right)+a\right)^2}{2(bS_\nu)^2}\right).$$

Integrating the resulting expression, we determine the damage distribution law per flight:

$$F_D(D)=\int_0^D f_D(D)\,dD=\Phi\left(\frac{\ln D-\ln\gamma\left(t^*\right)+a}{bS_\nu}\right),$$

where $\Phi(U)$ – probability integral:

$$\Phi(U)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{U}\exp\left(-\frac{t^2}{2}\right)dt.$$

Thus, for this model, the damage per flight is a log-normal random variable with a constant value damage logarithm dispersion $D[\ln D] = b^2(S^2_a + S^2_\sigma)$ and is dependent on the operating time of damage logarithm expectation $M[\ln D] = \ln\gamma(t^*) - a$.

From a computational point of view the simplicity and the clarity of the model due to the possibility of separation of expression (8) random and non-steady components of loading is obvious.

Let us consider a damage model under the condition of low-cyclic fatigue.

The rotor elements of compressors and turbines that are installed on modern aviation engines work under variable temperature modes within the material elasto-plastic limits, i.e. at the non-isothermal low-cyclic fatigue condition that causes plastic deformations and yield flow.

The destruction due to low-cyclic fatigue are typical for rotor discs, turbine blades, shafts and other rotor elements.

Non-elastic deformations change the dimensions of the elements and influence the material property of resisting repeated loadings.

Yield deformation influences the loading redistribution in the elements even in case of insignificant plastic deformations. That is why these deformations should be taken into account during element strength calculation.

In design practices the most popular methods of plasticity and yield ability analysis are the theories of deformation, yield flow ability and hardening.

Low-cyclic non-isothermal loading of gas-turbine engine hot section elements is caused by the frequent engine starts, cut off sand mode change during the engine operation cycles which are accompanied by the time-exposure at the different flight modes under the continuous loadings and temperatures. The calculation of a material's ability to resist low-cycle fatigue by means of the direct experiment method at such complex loading patterns practically impossible. That is why it is more reasonable to modify the loading calculation programs to simpler equivalent low-cycle modes of loading for which typical experimental characteristics of material endurance to low-cyclic loadings can be derived. Isothermic deformation characteristics are the limits of schematisation.

Prior to active loading σ(*t*) changing of operational program reduction to an equivalent one should be decomposed into low-cyclic $\sigma_m(t)$ and static $\sigma_{st}(t)$ components. Cyclic components $\sigma_m(t)$ can be subtracted by the method of complete cycles, "fall of rain" or another method starting from the initial cycle in which stress changes from zero to absolute maximum $\sigma_{\text{мах}}$ which corresponds to the full strength engine operation mode. Additionally, the initial cycle by the "fall of rain" method can determine elementary cycles with their minimal and maximal values (amplitude and average) of loadings per cycle.

Damages in every elementary cycle are estimated independently and are then summarised. In this case, according to the linear hypothesis of damage summation, the general damage accumulated during the flight can by calculated by the formula

$$\mathrm{D_{CC}} = \frac{1}{\mathrm{N_{des}}} + \sum_{i=1}^{m}\frac{1}{\mathrm{N_{desi}}} = \frac{1}{\mathrm{N_{des}}(\mathrm{T_{max}}, \sigma_{\mathrm{m}}, \Delta\varepsilon)} + $$
$$+\sum_{\mathrm{i=1}}^{\mathrm{m}}\frac{1}{\mathrm{N_{desi}}(\mathrm{T_{max}}, \sigma_{\mathrm{m}}, \Delta\varepsilon_{\mathrm{i}})},$$

where N_{des} N_{desi} – number of cycles till the change of the base and elementary cycles;

m – number of elementary cycles in the base cycle set.

Damage at one loading cycle for the simplest case of component stressed by the random cyclic loadings σ_i, which are governed by the same law of distribution $F(\sigma)$ according to linear hypothesis, can be calculated by the formula

$$\mathrm{D} = \frac{1}{\mathrm{N}(\sigma, \alpha_\tau)}$$

where σ – random loadings which arise during flight cycle ($M[\sigma] = \overline{\sigma},\ D[\sigma] = \mathrm{S}_\sigma^2$);

α_τ – centered random variable, which characterises material dispersion properties ($M[\alpha_\tau] = 0$, $D[\alpha_\tau] = \mathrm{D}_{\alpha\tau}^2$);

$N(\cdot)$ – material endurance function, by which number of cycles made before element destruction under the continuous cyclic loadings σ can be determined.

Quasi-static destruction takes place due to accumulation of one-sided plastic deformations, which are the result of low-cyclic loading. Such deformations are equal to single loading static deformations.

Destruction with crack formation occurs due to the accumulation of fatigue damages.

If local operational loads in the material caused by strength loading are determined experimentally or by solution of elastic or elasto-plastic problem, than according to fatigue destruction criteria (rigid-type stressing) and regardless of steel cyclic properties, the loadings σ_a^* destructive amplitudes for the construction with a given number of cycles before destruction *N* can be determined by Manson's formula (Manson 1966; Мэнсон 1974).

$$\sigma_{\mathrm{a}}^* = \frac{\mathrm{E}}{4\cdot \mathrm{N}^{\mathrm{m}_p} + \frac{1+\mathrm{r}^*}{1-\mathrm{r}^*}} ln\frac{100}{100-\Psi} + \frac{\sigma_{-1}}{1+\frac{\sigma_{-1}}{\sigma_{\mathrm{B}}}\left(\frac{1+\mathrm{r}}{1-\mathrm{r}}\right)} \quad (12)$$

where *E* –elasticity;

m_p – index of steel strength characteristic;

r^*, r – steel ratio coefficient of elastic and actual loading conditions correspondingly;

Ψ – relative reduction of investigated sample cross-section area;

σ_{-1} – endurance limit at the symmetrical cycle of loading (tension-compression);

σ_{B} – ultimate loading limit.

Endurance curve (12) may be expressed with respect to number of cycles before destruction by the following method:

$$\mathrm{N} = \left(\frac{\mathrm{a}_0}{\sigma_{\mathrm{a}}^* - \mathrm{a}_1} - \mathrm{a}_2\right)^{-1/\mathrm{m_p}}, \quad (13)$$

where:

$$a_0 = \frac{1}{4}E\ln\left(\frac{100}{100-\Psi}\right),$$

$$a_1 = \frac{\sigma_{-1}}{1+\frac{\sigma_{-1}}{\sigma_B}\left(\frac{1+r}{1-r}\right)},$$

$$\mathrm{a}_2 = \frac{1}{4}\left(\frac{1+\mathrm{r}^*}{1-\mathrm{r}^*}\right).$$

In order to create the description of strength characteristics it is necessary to add to the abovementioned model (13) parameters α_τ, which characterise possible material properties.

In case of low stress relation of mathematical expectation relation between cycle numbers and loading (13), the elements endurance stochastic characteristics can be described by means of three regularities:

$$N = \left(\frac{a_0}{\sigma_a^* + \alpha_\tau - a_1} - a_2\right)^{-1/m_p} \quad (14)$$

$$N = \left(\frac{a_0}{\sigma_a^* - a_1} - a_2\right)^{-1/m_p} \cdot e^{\alpha_\tau} \quad (15)$$

$$N=\left(\frac{a_0}{\sigma_a^*+\alpha 1_\tau-a_1}-a_2\right)^{-1/m_p}\cdot e^{\alpha 2_\tau}. \qquad (16)$$

Introduce S_α^2. In equation (14) this parameter determines uniform diffusion of strength characteristics around the stress logarithm $\ln(\sigma_a)$, while in model (15) around the logarithm of cycle number ln*(N)*. Strength characteristic model (16) in general case includes two dependent random variables with different dispersions $S_{\alpha 1}^2$, $S_{\alpha 2}^2$ and $M[\alpha 1]=M[\alpha 2]=0$ determines the characteristics of diffusion around the endurance curve.

The general scheme of component's loadings is described on figure 6.

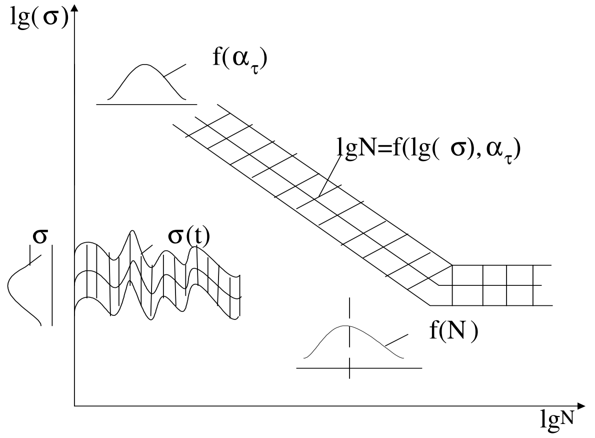


Fig. 6. The general scheme of component stressing and probability strength characteristics

Endurance curve $\sigma_a(N)$ for the element made of material 08X18H10T under the symmetrical loading cycle and endurance probability model $\sigma_{a1}(N,\alpha_\tau)$, $\sigma_{a2}(N,\alpha_\tau)$ constructed in accordance with the formulas (6) and (7) is shown in figure 7.

Curves $\sigma_{\alpha 1}(N)$ и $\sigma_{\alpha 2}(N)$ correspond to fractals $\overline{\sigma_a(N)}\ \pm 3S_\alpha^2$.

Steel characteristics and loading cycle parameters correspond to the manufacturer calculation:

E= 205000 MPa,

Ψ =42,5, $\sigma_B=491$ MPa,

$\sigma_{-1}=0,4\sigma_B$ =196 MPa,

$r^*=r=\dfrac{\sigma_{min}}{\sigma_{max}}$ =0.

In a simpler case for different material function $N(\cdot)$ can be described by low strength relations

$$N=\begin{cases}C(\sigma+\alpha_\tau)^m, & \sigma>\sigma_{-1}\\ \infty, & \sigma<=\sigma_{-1}\end{cases}, \qquad (17)$$

or

$$ln(N)=\begin{cases}C_1+m\,ln(\sigma+\alpha_\tau) & \sigma<=\sigma_{-1}\\ \infty, & \sigma<=\sigma_{-1}\end{cases}, \qquad (18)$$

where $C_1=ln(C)$

$$N=\begin{cases}C\,\sigma^m e^{\alpha_{\tau 2}}, & \sigma>\sigma_{-1}\\ \infty, & \sigma<=\sigma_{-1}\end{cases}, \qquad (19)$$

$$N=\begin{cases}C(\sigma+\alpha_{\tau 1})^m e^{\alpha_{\tau 2}}, & \sigma>\sigma_{-1}\\ \infty, & \sigma<=\sigma_{-1}\end{cases}. \qquad (20)$$

Within the error limit, corresponding models (14)-(15) and (16)-(20) are approximately equal (Fig. 8). The results of calculation are shown on figure 9.

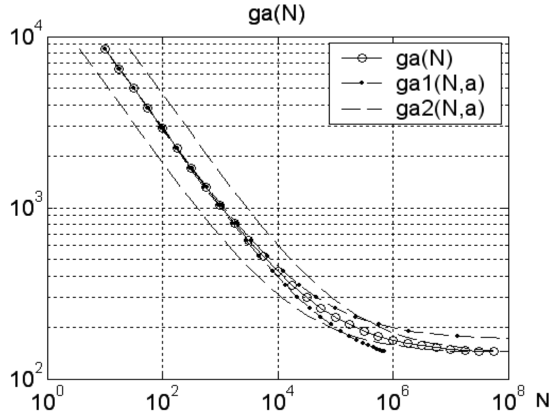


Fig. 7. Endurance characteristics probability model of steel 08X18H10T

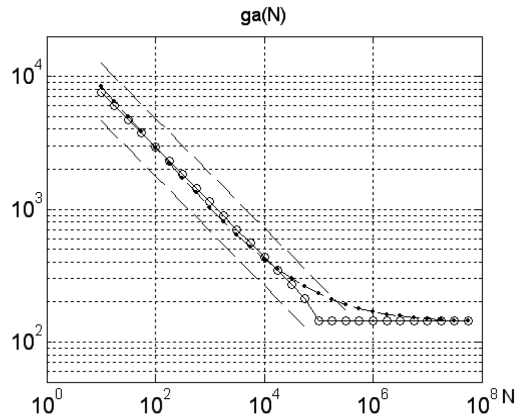
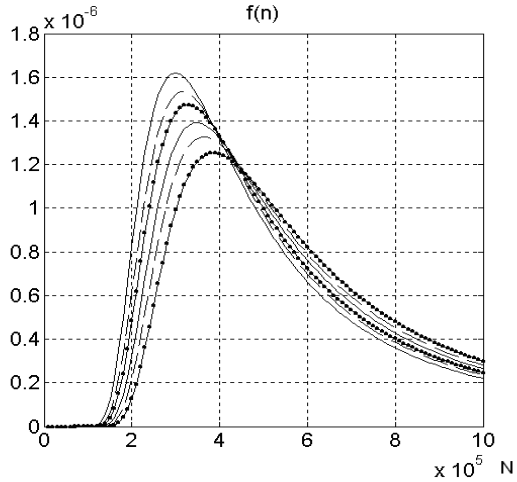
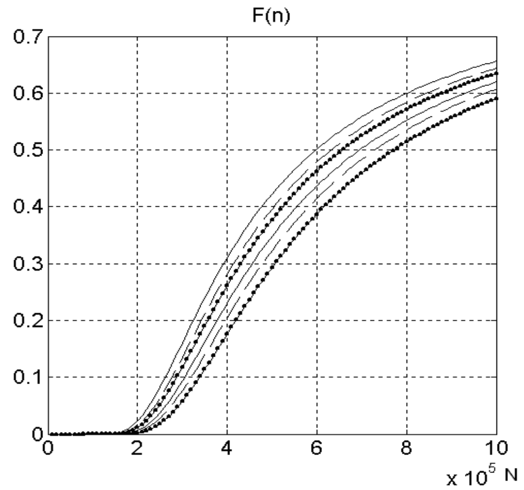


Fig. 8. The general and linear fatigue endurance model of steel 08X18H10T



a)



b)

Fig. 9. Densities (*a*) and distribution function (*b*) of cycle number before element destruction

5. The definition of probabilistic characteristics for loading and capacity of turbine engine component service life

The service life in the real environment allows the use of more complicated configuration's including stock performance components and will provide benefits while maintaining high reliability in service.

Design safety margins of engine vital parts, which include rotating blades, blast wheels, compressor and turbine wheels and rotor shafts, are the initial basis for evaluating the permissible operating time in any method of establishing a service life (fixed, differential). The calculation of depletion and monitoring of service life and other strength integrity characteristics is carried on the basis of loading characteristics estimation of structural elements on all modes of turbine engine operating cycle and by using standard characteristics of constructional material strength (such as durability, high-cycle and low-cycle fatigue) through the accumulated damage characteristics.

For the comparative estimation of turbine engine part structural strength we use certain strength factors that determine their strain, deformability, carrying capacity and longevity. For strength the factors of safety k are compared with the lowest bearable factors of safety $k_{\min}$ and in the case of inequality $k > k_{\min}$ the strength margin for the concerned parameter meets the strength standards.

One of the major criteria of turbine engine part structural strength envisage is the strength factor.

$$\mathrm{k}_{\sigma} = \frac{\sigma_{\mathrm{lim}}}{\sigma_{\mathrm{eq}}},$$

where σ_{lim} – limit load, which characterises limits of material; σ_{eq} – equivalent net loads.

The limit and operating load calculation depends on the part's job conditions. The ultimate load σ_{B} is taken as σ_{lim} and the highest tensile loadings σ_{max} are taken as σ_{eq} under static loadings.

In the case of complex loads, lumped stresses are calculated by the strength models (Биргер *и др.* 1979; Кучер 1985). The long-term strength σ_{dur} is taken as the limit load under service conditions at high temperatures and constant or slowly varying loads when material properties change continuously, and for variable symmetric cyclic loads it is the fatigue point σ_{-1}.

In this case the expressions for strength factors have the following structure:

$$\mathrm{k}_{\sigma\mathrm{dur}} = \frac{\sigma_{dur}}{\sigma_{eq}}, \quad \mathrm{k}_{\sigma\mathrm{a}} = \frac{\sigma_{-1}}{\sigma_{eq}}.$$

Durability is chosen appropriate to the time t and temperature T and the fatigue point is chosen appropriate to the number of cycles N and temperature T.

Strength factors are expressed in complex formulas for the joint action to re-static and high mechanical and thermal-cycle stresses or their pair wise sets (Расчет... 1984). These coefficients are in better agreement with experimentation but their application is connected with the necessity to conduct special research.

Besides strength factors for the stress to assess the structural strength of turbine engine parts we use the safety margin of longevity

$$\mathrm{k}_{\tau} = \frac{\tau(\sigma_{eq})}{\mathrm{t}}, \tag{21}$$

where $\tau(\cdot)$ – time to failure at load equal to the equivalent σ_{eq}; t – load operating time.

Safety factors of turbine engine vital parts are analysed basing on the correlation of design and experimental values gotrom the operation values in engineering practice. Specified supplies are usually set for blades, which are designed to operate in certain conditions and produced from this material. In this case safety factors are peculiar similarity criteria of parts and can be set depending on the results of parts tests of this type.

The strength factor and durability factor are considered under service condition as dynamic characteristics that vary depending on operating time and previous load history.

Monitoring of these stress characteristics is better to envisage through the damage part characteristics using the linear hypothesis of static damage summation by bringing stressing capabilities to an equivalent operation.

Let us consider the concept of rate definition $\mathrm{k}_{\sigma\mathrm{dur}}$ and k_{τ} under operation.

Every engine operating mode j is characterised by certain level of acting loading in structural element σ_j and temperature T_j. The long-term static damage rate of structural elements on j according to the linear hypothesis of static damage summation equals relative operating period on this operation

$$\mathrm{D_j} = \frac{\mathrm{t_j}}{\tau(\sigma_{\mathrm{j}}, \mathrm{T_j})}, \tag{22}$$

where $\mathrm{D_j}$ – operation's damage; t_j – operation's durability; $\tau(\cdot)$ – material life characteristics.

For the equivalent operation with parameters t_{ej}, σ_e, T_e the damage D_{ej} could be found by a similar formula

$$D_{ej} = \frac{t_{ej}}{\tau(\sigma_e, T_e)}. \quad (23)$$

According to the equivalence condition $D_j = D_{ej}$ equivalent operation must create the same damage as given above Therefore equivalent operation durability can be determined by the ratio

$$t_{ej} = \frac{t_j \tau(\sigma_e, T_e)}{\tau(\sigma_j, T_j)}. \quad (24)$$

The total equivalent operation durability for some i^{th} flight is determined by the expression

$$t_{ei} = \sum_{j=1}^{m_i} t_{ej}, \quad (25)$$

where m_i – the number of loading segments in flight i.

For n flights accumulated equivalent operation durability is expressed by the sum

$$t_e = \sum_{i=1}^{n} t_{ei}. \quad (26)$$

As an equivalent operation, one of the most intense operating modes is usually chosen. It is the maximum engine operating mode for the turbine blades.

According to the equation of the set of stress rupture curves of material structural elements $\tau(\sigma_e, T_e)$, using the equivalent time amount t_e to applicable temperature T, the equivalent long-term strength $\sigma_{eD}(T_e, \sigma_e)$ can be found and the accumulated equivalent durability factors of strength are determined by the formula

$$k_\sigma = \frac{\sigma_{eD}(T_e, t_e)}{\sigma_e}. \quad (27)$$

In order to obtain an effective formula for k_σ it is necessary to provide the possibility of a solution for an equation of the strength curve with respect to stress and durability variables. Otherwise, the equivalent long-term strength limit should be calculated by the iterative method. The exponential and strength equation can be solved by the direct calculation method. For such equations we will consider the methods of long-term-strength safety factors estimation

When material long-term-strength characteristics are described by means of the exponential expression

$$lg\tau = A(T) + B(T)\sigma, \quad (28)$$

according to formula (27) the strength safety factor can be calculated in the following way:

$$k_\sigma = \frac{lg\ t_e - A(T_e)}{B(T_e)\sigma_e}, \quad (29)$$

where $A(T), B(T)$ – temperature function.

Taking into account expressions (22)...(29), the strength safety factor can be express in terms of accumulated element damage D_n during n flights

$$k_\sigma = \frac{lg \sum_{i=2}^{n} \sum_{j=1}^{m_i} \frac{\tau(\sigma_e, T_e)}{\tau(\sigma_j, T_j)} t_j - A(T_e)}{B(T_e)\sigma_e} = 1 + \frac{\ln D_n}{\eta}, \quad (30)$$

where

$$D_n = \sum_{i=1}^{n} D_i = \sum_{i=1}^{n} \sum_{j=1}^{m_i} D_{ij}, \quad (31)$$

$\eta = ln(10) B(T_e)\sigma_e$.

Using the following strength characteristics

$lg\tau = A(T) + B(T) lg\sigma$, (or $\tau = C(T)\sigma^{B(T)}$,

where

$C(T) = 10^{A(T)}$)

dependence of strength safety factor k'_σ on accumulated damage is expressed by the formula

$$k'_\sigma = \frac{1}{\sigma_e} \left(\frac{1}{C(T_e)} \sum_{i=1}^{n} \sum_{j=1}^{m_i} \frac{\tau(\sigma_e, T_e)}{\tau(\sigma_j, T_j)} t_j \right)^{\frac{1}{B(T_e)}} = D_n^{\frac{1}{\vartheta}}, \quad (32)$$

where

$$\vartheta = B(T_e).$$

Using the expressions (21), (22), (31), the equivalent durability capacity which is determined as a variable opposite to accumulated damage can be found

$$k_D = \frac{1}{D_n}. \quad (33)$$

Taking into account expressions (30), (32) and (33) at the equal level of accumulated damages the dependence between minimal normalised durability safety factors $k_{D\min}$ and strength $k_{\sigma\min}$, $k'_{\sigma\min}$ can be established:

$$k_{\sigma\min} = 1 - \frac{\ln k_{D\min}}{\eta}, \quad (34)$$

$$k'_{\sigma\min} = \left(k_{D\min}\right)^{-1/\vartheta}. \quad (35)$$

Equalities (34) and (35) normalise and reduce to common criteria of critical safety factor values for stresses or durability for different types of strength characteristic descriptions.

For the investigation of the quantitative regularity of long-term strength reduction process during the operation the service life reduction factor is often applied This factor is also assumed as proportional to equivalent working time of an element or number of flight cycles n:

$$k_B = \frac{n}{n_{cr}} 100\ \%,$$

where n_{cr} – critical number of flight cycles before full life exhaustion.

When k_D is used as a criterion of strength, then life reduction factor will be proportional to the accumulated damage:

$$k_{BD} = \frac{k_{D\min}}{k_D} 100\ \% = k_{D\min} D_n 100\ \%.$$

In this case of the critical number of cycles before full life exhaustion we can write the following:

$$n_{cr0} = \frac{n}{k_{BD}} 100\ \% = \frac{n \cdot k_D}{k_{D\min}} = \frac{n}{k_{D\min} D_n}.$$

6. Conclusions

In this research models and damage calculation methods for damage, remaining durability, airframe structural elements and engine life time exhaustion in accordance with the criteria of long-term durability, low-cycle and high-cycle endurance, also for elements with cracks caused by the one-sided, multi-sided and multi-component loading are investigated in determinate and probable variants under continuous loading and various stress conditions.

Suggested models can be used during the calculation of possible types of destruction probability, choice of optimal constructive and operational work that permits an increase in component stress resistance capacity. This, as a rule, is a result of analysis and compromise between different types of loading and also difficulties from its result and lifetime technically and economically reasonable.

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ORLAIVIŲ KONSTRUKCINIŲ ELEMENTŲ STIPRUMO PATIKIMUMO SAVYBIŲ SKAIČIAVIMO MATEMATINIAI MODELIAI

M. Kulyk, O. Kucher, V. Miltsov

S a n t r a u k a

Darbe išanalizuoti pažeidimų matematiniai modeliai, kai veikia statinės ir ciklinės apkrovos. Išnagrinėti pažeidimų modelių parametrai bei ilgaamžiškumo savybės, reikalingos ilgalaikio stiprumo išnaudojimui, mažacikliam ir daugiacikliam nuovargiui, termocikliniam ilgaamžiškumui ir takumui įvertinti. Apibendrinti modeliai, įvertinantys daugiaašių ir daugiakomponenčių apkrovų ypatybes. Pateikti pažeidimų modeliai, esant apkrovų spektrui, tikimybiniai stiprumo ir pažeidimų kaupimo modeliai, esant atsitiktiniams poveikiams. Išanalizuoti resurso išnaudojimo kriterijai, parodytas jų ryšys su sukauptais pažeidimais.

Reikšminiai žodžiai: ilgalaikis stiprumas, mažaciklis nuovargis, daugiakomponentė apkrova, daugiaciklis nuovargis, pažeidimų modelis, sudėtingų įtempimų būsena, apkrovų spektras, ištvermingumas įtrūkiams.