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# RELIABILITY OF FLEET OF AIRCRAFT TAKING INTO ACCOUNT INFORMATION EXCHANGE ABOUT THE DISCOVERY OF FATIGUE CRACKS AND THE HUMAN FACTOR

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**Abstract.** The importance of using information about the discovery of fatigue cracks to eliminate any fatigue failure in a fleet of aircraft of the same type and the human factor influence is studied. Numerical estimation of the influence of this information exchange is obtained. The Monte Carlo method was used for modelling the process of inspecting the aircraft fleet and calculating the probability of fatigue failure as a function of the inspection interval, number of aircraft in the fleet, and intensity of the process of bringing new aircraft into operation.

**Keywords:** Monte Carlo, inspection programme and approval test, fleet reliability.

## 1. Introduction

Review of the problem of eliminating aircraft fatigue failure can be seen in (Paramonov *et al.* 2011). General approaches to the inspection programme problem are discussed by F. Beichelt and P. Franken, and by I. Gertsbakh (Beichelt, Franken 1983; Gertsbakh 2000). Models of fatigue crack are discussed by J. N. Yang and S. D. Manning (Yang 1980; Yang, Manning 1980).

Our paper is a development of papers in (Paramonov 1999; Paramonov, Kuznetsov 2009, 2008, 2006; Paramonov, Hauka 2010a, 2010b; Hauka, Paramonov 2010).

The problem is solved by using two main methods: by discarding aircraft after a specified number of flight hours is reached (safe-life approach) or by implementing periodic inspection programmes that allow the fatigue damage to be found and repaired or that allow an aircraft to be removed from service before the damage exceeds the regulatory mandated value (fail-safe approach). The main approach used today is the development of an inspection programme that is developed using information about full-scale aircraft fatigue test.

In this paper we consider a solution to this problem provided that the cumulative distribution function (*cdf*) of time to failure is known. We assume that some structurally significant item (SSI), the failure of which is the failure of the aircraft, is characterised by a random vector (r.v.) ($T_D$, $T_C$), where $T_C$ is a critical lifetime (up to failure) and $T_D$ is a service time when some damage (fatigue crack) can be detected (with probability equal to unit). So if there are N aircraft in the fleet and *r* inspections are conducted at the interval ($T_D$, $T_C$) then the probability of discovering a crack will be equal to $F_e = 1-(1-w)^r$, where *w* is probability of detecting a crack, which depends on the quality of the regular inspections (human factor). We also suppose that the required operational life of the system is limited by specified life (SL), $t_{SL}$, when the aircraft is removed from service.

Calculations of the probability of fatigue failure should be based on a model of fatigue crack propagation and the processing of the corresponding fatigue test result. As shown in (Paramonov *et al.* 2011), the following simple exponential model of fatigue crack could be used for approximating its dependence on time:

$$a(t) = a_0 e^{Qt},$$

where *a(t)* is the size of a fatigue crack at time *t* (the number of flight hours), $a_0$ is the so-called equivalent initial crack size, and Q is a parameter that depends on the loading mode. Model parameter estimates are derived from the full-scale fatigue test of the aircraft using regress analysis as follows:

$$a(t) = a_0 e^{Qt}; \qquad (1)$$

$$\log a(t) = \log a_0 + Qt; \qquad (2)$$

$$\text{thus} \begin{bmatrix} \log a(t_1) \\ \log a(t_2) \\ \dots \\ \log a(t_n) \end{bmatrix} = \begin{bmatrix} 1 \; t_1 \\ 1 \; t_2 \\ \dots \\ 1 \; t_n \end{bmatrix} \times \begin{bmatrix} \log a_0 \\ Q \end{bmatrix}, \qquad (3)$$

where pairs of $\{(t,a)_i$, $i$ = 1,…, n} are the results of tests (observations of real cracks). For a numerical example, we accept that $\alpha = 0.286132583$, but $\ln(Q)$ is a random variable (r.v.) that has normal distribution. Its mean value and standard distribution are equal to –8.58733 and 0.155689 (Paramonov *et al.* 2011). As soon as we have model parameters $a_0$ and Q, we can perform crack modelling.

## 2. Development of inspection programme for one aircraft

If there is only one aircraft, then the probability of the failure of that kind of fleet would be equal to the probability of the failure of that single aircraft. To develop an inspection programme for that aircraft, we use the Monte-Carlo method. We suppose that r.v. $T_D$ and $T_D$ are defined by r.v. $Q$:

$$T_D = \frac{\log a_d - \log a_0}{Q} = \frac{C_d}{Q}; \qquad (4)$$

$$T_C = \frac{\log a_c - \log a_0}{Q} = \frac{C_c}{Q}, \qquad (5)$$

where $a_d, a_c$ are a detectable and critical size of fatigue crack.

Let us simulate one thousand times the development of the crack on a given aircraft and calculate the function of failure probability on the interval between the inspections for different values of w (w = 0.9 and w = 0.95). In a general case, the choice of time of first inspection, $t_1$, should have a specific base, but in this paper, for the purpose of simplicity, we suppose that all inspection intervals are equal. The random probability of the failure of a specific aircraft with specific random $T_d = t_d$, $T_c = t_c$ is:

$$P_{f1} = (1-w)^R, \text{ if } T_c < t_{SL};$$

$$P_{f1} = 0, \text{ if } T_C > t_{SL}, \qquad (6)$$

where $R$ is a random number of inspections in the period of time between $T_D$ and $T_C$, specifically:

$$R = r = \left[\frac{t_c}{D}\right] - \left[\frac{t_d}{D}\right], \qquad (7)$$

where $[x]$ is integer part of $x$ and D is the inspection interval. It is worth remembering that for the case when $T_C > t_{SL}$, even though a crack was not found, the SSI will be taken out of service before the failure happens. In that case, the probability of failure is equal to zero.

In our example, it was assumed that $t_{SL} = 42000$. Value $P_{f1}$ is function of Q. So its mean value is:

$$P_{f1} = \int_{-\infty}^{\infty} p_{f1}(q)dF_Q(q), \quad (8)$$

where $F_Q(q)$ is the cumulative distribution function of Q. As already stated, for the calculation of $P_{f1}$ we have used the Monte Carlo method.

In figure 1 the failure probability as a function of the inspection interval for one aircraft is given.

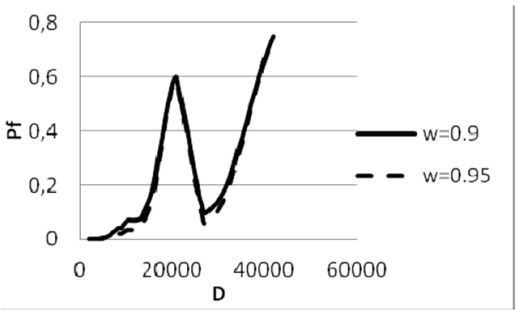


**Fig. 1.** Dependency of failure probability on inspection interval for one aircraft

We can see that for small $P_f$ with the increasing inspection interval the probability of failure increases. With less qualitative inspection cases, $w_1 < w_2$, the failure probability with the same interval will be higher. We can also see some interval of non-monotonic behaviour of function that is expressed by a sharp decrease in the probability of failure. This phenomenon is explained in (Paramonov *et al.* 2011), but we should make the choice of D in the first interval with small $P_{f1}$. As an example, we develop the inspection programme (make choice of D) for an allowable level of fatigue failure $P_{f\max} = 0.05$. So, if w = 0.9, then D = 9360, but if w = 0.95, then D = 13430, where D is the interval between the inspections.

## 3. Fleet without information exchange

In a fleet without information exchange, the results of aircraft inspections are not used for aircraft SSI failure prevention of other aircraft of the same kind. All aircraft are inspected independently. If the fleet consists of N aircraft, then the probability of failure of at least one aircraft in that fleet is equal to:

$$p_{fN} = 1-(1-p_{f1})^N. \quad (9)$$

With an increase in the quantity of aircraft in a fleet, the probability of failure increases. It depends on $p_{f1}$ and N (Fig. 2).

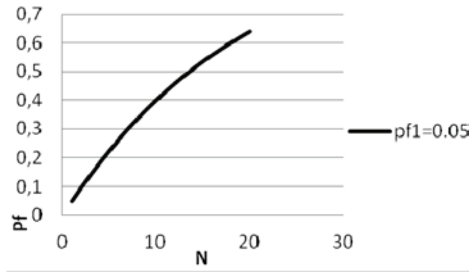


**Fig. 2.** Dependency of failure probability on number of aircraft in fleet without information exchange

To keep this probability under the specified level, specific inspection programmes should take the size of the fleet (number of aircraft in fleet) into account during development (Fig. 3).

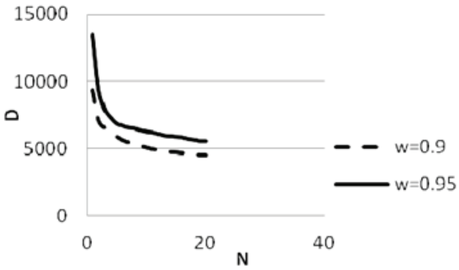


**Fig. 3**. Dependency of inspection interval on number of aircraft in fleet without information exchange for constant failure probability ($p_{fN} = 0.05$)

## 4. Fleet with information exchange. All aircraft begin service simultaneously

To prevent failure in a fleet with information exchange, it is enough to find at least one crack before the failure of any aircraft in the fleet. The random probability of this event is found by the formula: $P_{fNW} = (1-w)^R$, where R is the total random number of inspections before specific $t_{SL}$ of every aircraft and before first failure in the entire fleet. Mean value of $P_{fNW}$ can be calculated in following way. We suppose that i$^{th}$ aircraft begin service at 'calendar' time moment $t_i = (i-1)\Delta T, i = 1,...,N$. In this paper, we suppose that $\Delta T$ is some constant (in general it can be, for example, the random time interval between the events in some Poisson process).

Let $T^+_{d_i} = t_i + T_{d_i}$ and $T^+_{c_i} = t_i + T_{c_i}$ be the calendar time moments when a fatigue crack and correspondingly aircraft failure can be discovered. And let

$I_{SL} = \{i : T_{ci} < t_{SL}, i = 1,...,N\}$ be a set of indexes of aircraft, the failure of which can take place if inspection does not take place.

Define $T^+_f = \min\left\{T^+_{c_i} : i \in I_{SL}\right\}$,

$T^+_{fi} = \min(T^+_{c_i}, T^+_f)$, $i \in I_{SL}$, and finally

$$R = \sum_{i \in I_{SL}} R_i, \quad (10)$$

where $R_i = \max(R_{if} - R_{id}, 0)$, $i \in I_{SL}$;

$R_{if} = \max([(T^{+}_{fi} - t_i)/D], 0)$ , $R_{id} = [T_{di}/D]$ ,
is the random inspection number of i[th] aircraft from the set $I_{SL}$ for inspection interval D (a specific schedule of inspections for each aircraft is supposed: $t_i + D, t_i + 2D, \ldots$ ). Random variable $Q$ is the speed of fatigue crack growth in the logarithmic scale. It has a specific realisation for each aircraft, and $Q_1, \ldots, Q_N$ are independent random variables. The mean value of $P_{fNW}$ therefore is:

$$P_{fNW} = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \left((1-w)^{r(q)}\right) dF_{Q1}(q1) \ldots dF_{Q_N}(q_n), \quad (11)$$

where $r(q), q = (q_1, \ldots q_n)$ is realisation of random variable $R$. For the large number $N$, only the Monte Carlo method is appropriate for calculation of $p_{fNW}$ . A corresponding PC program was used for numerical examples in this paper. But first we suppose that all aircraft are brought into service simultaneously: ΔT = 0. Using the same values of $t_{SL}$ , w, and D, we obtain the results shown in figure 4.

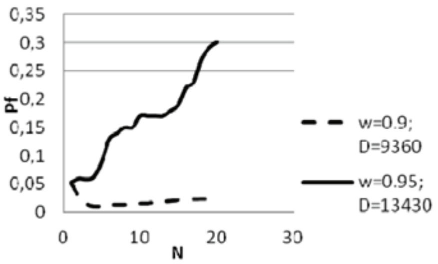


**Fig. 4.** Dependency of failure probability on number of aircraft in fleet with information exchange and ΔT = 0

We see that the probability of failure of any aircraft in the fleet with information exchange programmes is lower than in figure 2, although $p_{f1} = 0.05$ for both inspection programmes. For the inspection programme with the interval between inspections being D = 13430, the increase in failure probability with growth in the number of aircraft in the fleet is less intensive than for the fleet without information exchange. For the inspection programme with the interval between inspections being D = 9360, with growth in the number of aircraft in the fleet, the probability of failure in that fleet decreases. We can see a comparison of failure probabilities in figure 5.

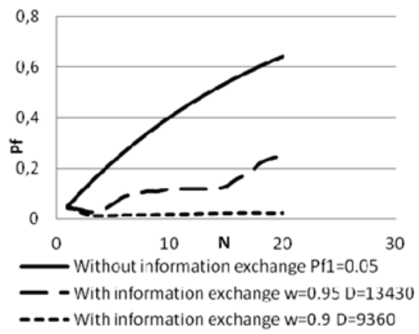


**Fig. 5.** Comparison of failure probabilities of fleet with and without information exchange for different number of aircraft at different w, D, and ΔT = 0

For the fleet without exchange of information, for $p_{f1} = 0.05$ the function of failure probability on N for w = 0.95/ D = 13430 and w = 0.9/ D = 9 360 are the same since they depend only on $p_{f1}$. The inspection programmes developed (choice of D) for those fleets are shown in figure 6.

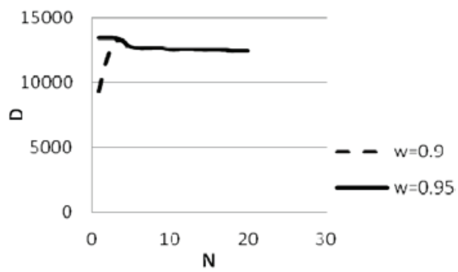


**Fig. 6.** Dependency of D on number of aircraft in fleet with information exchange for constant failure probability ($p_{fNw}$ = 0.05)

In figure 6 we can see that, with growth in the number of aircraft in the fleet, the effect of *w* on the choice of intervals between the inspections decreases; the change in intervals between the inspections are less intensive than for the fleet without information exchange.

**5. Fleet with information exchange. Different beginning of aircraft service**

In reality, cases when all the aircraft in a fleet are brought into service simultaneously do not take place. Now we suppose that new aircraft begin service after time interval ΔT after previous aircraft begin service. Let us see how the fleet's failure probability changes for different values of ΔT, ΔT > 0. First we are going to observe the case when w = 0.95 and D = 13430 for different ΔT. Results of calculations are shown in figure 7.

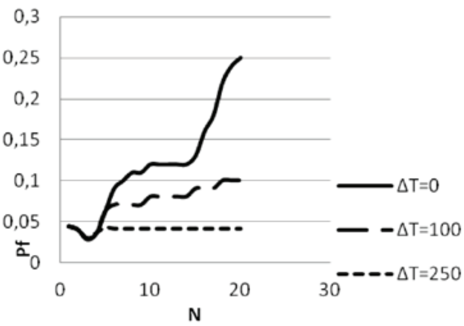


**Fig. 7.** Dependency of failure probability on number of aircraft in fleet with information exchange, for different ΔT for w = 0.95 and D = 13430

Now we see that if ΔT increases the failure probability decreases and tends to take a value of $p_{f1}$. In figure 8 we see inspection interval D needed to get $p_{fNW} = 0.05$ as function of fleet size, N, for two values: ΔT = 0 and ΔT = 250.

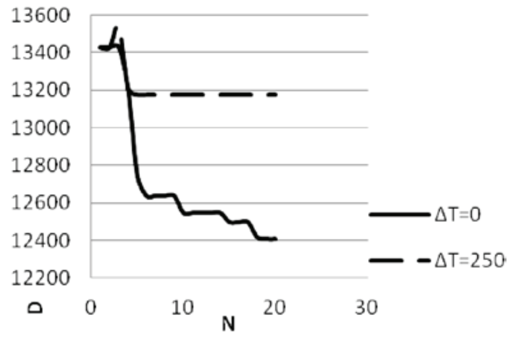


**Fig. 8.** Dependency of inspection interval for constant failure probability ($p_{fNw}$ = 0.05) on number of aircraft in fleet with information exchange, for w = 0.95 and different ΔT

More frequent inspections should be done when ΔT = 0. The comparison with the programme developed for the fleet without information exchange is shown in figure 9.

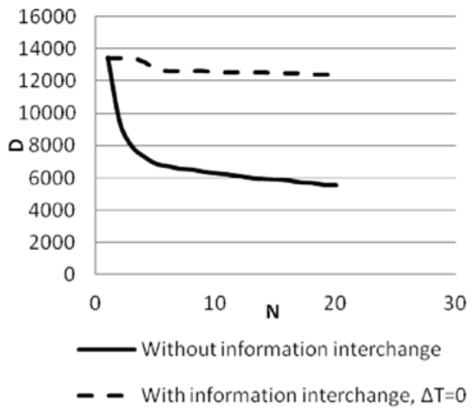


**Fig. 9.** Dependency of inspection interval in fleet with and without information exchange on size of fleet for constant failure probability ($p_{fNw}$ = 0.05) and w = 0.95

This comparison shows that approximately two times more inspections for the fleet without information exchange should be made for the same fleet reliability.

Now let us study how the fleet's failure probability changes when w = 0.9 and D = 9360. Corresponding functions for different values of ΔT are represented in figure 10.

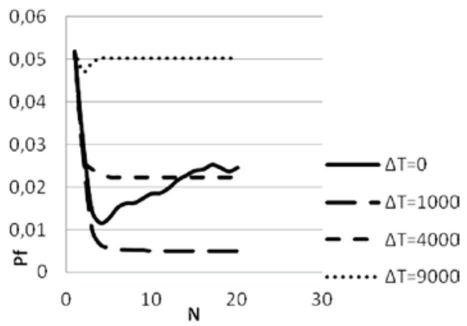


**Fig. 10.** Dependency of failure probability on number of aircraft in fleet with information exchange, for different ΔT, w = 0.9 and D = 9360

It is shown that with increasing ΔT the failure probability tends to take a value of $p_{f1}$ again.

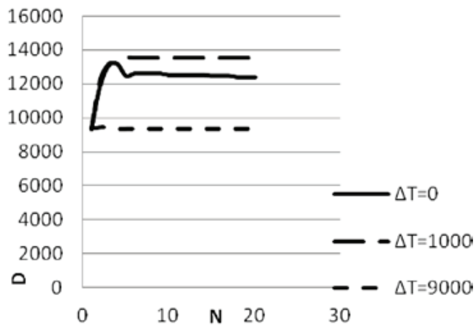


**Fig. 11.** Dependency of inspection interval on number of aircraft in fleet with information exchange for different ΔT and w = 0.9, for constant failure probability ($p_{fNw}$ = 0.05*)*

In figure 11 it is shown that more frequent inspections have to be done when ΔT = 9000. The comparison with the programme developed for the fleet without information exchange is shown in figure 12.

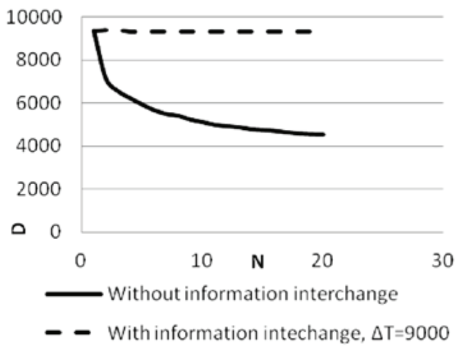


**Fig. 12.** Dependency of inspection interval in fleet with and without information exchange on size of fleet for constant failure probability ($p_{fNw}$ = 0.05) and w = 0.9

Again we see that the intervals between inspections tend to differ approximately in two times with the growth in the number of aircraft in the fleet.

## 6. Conclusion

In a previous investigation (Paramonov *et al.* 2011), the reliability of one aircraft was studied usually with recalculation for aircraft fleet reliability using equation (9) instead of (11) without taking into account information exchange about crack discovery and the human factor. Of course, it is obvious that information exchange is useful for increasing the reliability of a fleet of fatigue-prone aircraft and the human factor is very important, but in this paper a numerical estimation of these phenomena is obtained. A necessary method and a PC program are developed.

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