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CALCULATION OF THE CHARACTERISTICS OF A UAV LAUNCH FROM A RAMP

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Abstract. A launch from a ramp is applied in order to increase the effectiveness of unmanned aircraft. To attain this goal, the kinematic parameters of movement during a UAV launch from a ramp are analyzed, formulas to determine these parameters are formulated, and a method for calculating rational geometrical parameters of a ramp, which provides a safe lift-off of a UAV with the specified flight data, is used. This paper considers the advantages of a UAV launch from a ramp and the development of algorithms that help to calculate the characteristics of a UAV launch from a ramp.

Keywords: unmanned aerial vehicle (UAV), launching devices, ramp, trajectory of a lift-off, energetic height, thrust-to-weight ratio.

1. Introduction

Lately, the rapid development of unmanned aircraft has considerably expanded the list and content of relevant problems (Iliushko *et al.* 2009; Czerwiński *et al.* 2014). The application of special ramps can be one of the actions taken to increase the effectiveness of unmanned aerial vehicle application under the condition that there islack of platforms required for their launch (Miller 2009; Kennedy 2006). The application of UAVs is effective when launched from decks of ships, and also from specially equipped platforms in mountainous and woody areas. A ramp in stationary conditions usually has a curvilinear surface providing the optimum launch. Launching devices located on the gear of a vehicle have higher manoeuvrability. In this case, rectilinear directions, according to which a UAV lifts off, are installed at a particular angle towards the ground surface. A launching device, in all cases, provides a UAV launch with more useful loading (a sufficient fuel supply) and, consequently, increases its fighting capabilities. Special ramps for the launch of piloted aerial vehicles were successfully applied since the beginning of the 90's, first of all, to meet the needs of navy military aircraft. Their application has allowed reducing the speed of a lift-off from a surface, reducing the length of running start, and increasing the weight of useful loading onboard. It is obvious due to the advantages of a ramp that it is expedient to use them for unmanned aircraft, taking into account their specific features, which demands the development of special methods and procedures to calculate the characteristics of a launch from a ramp for particular UAVs.

2. Calculation of geometric parameters of a ramp

The ramp is a small inclined platform which is a continuation of the horizontally located part of a runway (Fig. 1). The inclined platform can be flat or curvilinear. A launch from a ramp has two significant capabilities unlike a usual take-off. First, owing to small speed, the lifting force during this kind of launch is lower than the gravitational force at the moment of a lift-off. Secondly, a UAV lifts-off from a ramp with some initial gradient angle of a climb trajectory, which is equal to the ramp elevation angle at its final point 2 (Fig. 1).

After lifting off, a UAV moves on a trajectory close to a ballistic one, at first, gaining height (to point 3), and then (depending on the value of its thrust-to-weight ratio) it can move down with an increase in speed to a safe value.

The launch ought to be arranged in order to have a necessary stock (reserve) of height when descending at point 4 and speed providing the safe continuation of flight. Safe height and speed depend on both aircraft characteristics (speed of a launch from a ramp V_2 its thrust-to-weight ratio ($\mu = P/G$, where $G = gm$) and bearing properties of a wing (value of C_{ymax})) and ramp characteristics (its length, climb angle and traction of the booster). Thus, ramp characteristics place certain requirements on UAVs, in particular regarding their weight, size, flying and operational characteristics (Silkov 1997). Variations of speed during the running start on a ramp surface are included in equation (1):

$$j_x = \frac{dV}{dt} = g(n_{xa} - \sin\theta),$$

$$\text{where } n_{xa} = \frac{P - X_a - \sum_{i=1}^{2} F_{f_i}}{G}. \quad (1)$$

Taking into consideration that the attack angle remains constant during the running start of a UAV, the difference between friction forces (F_f) and front resistance forces (X_a) is approximately constant. Consequently, the value of a tangential overload n_{xa} is close to a constant value as well (Silkov 1997).

From mechanics, it is known that for accelerated movement in regular intervals the length of a passable distance of a horizontal part (L_{0-1}) at a final speed (at point 1) is connected by the following dependence:

$$L_{0-1} = \frac{V_1^2}{2jcp}; \quad (2)$$

$$j_{cp} = gn_{xa} = g(\mu_x - f_{rf}), \quad (3)$$

where $\mu_x = \frac{P_x}{G}$ is the average thrust-to-weight ratio of a UAV;

j_{cp} is the average acceleration in a movement direction;

$f_{rf} = 0.5(f + \frac{1}{k_1})$ is the resulting friction factor.

The resulting friction factor considers the resistance of a UAV's movement from a take-off surface (through

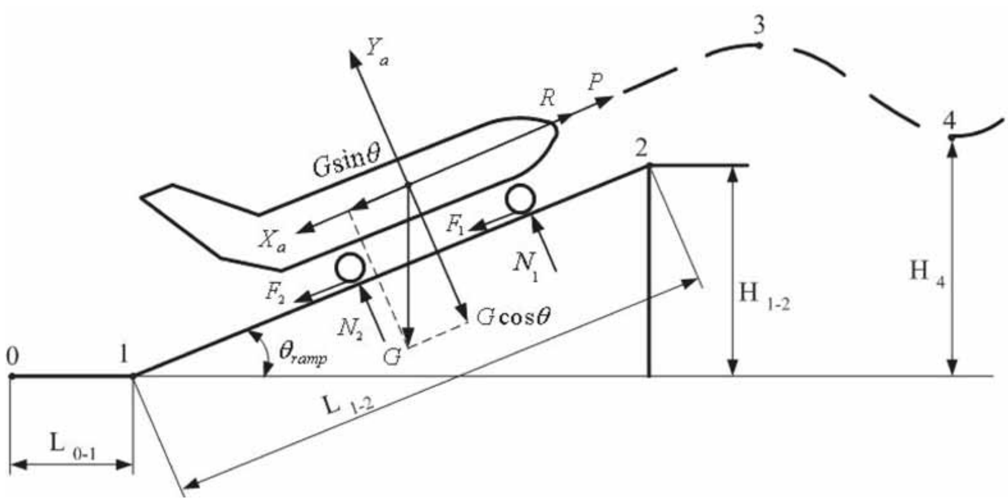

Fig. 1. Model of a ramp

friction factor f) and from the air environment (through aerodynamic quality at the end of the horizontal part k_1).

Where the horizontal part ends, the ramp actually begins. Its surface can be flat (Fig. 1) or curvilinear in accordance with radius r (Fig. 2).

Thus, the height ($H_{1\text{-}2}$) of a ramp is limited to constructive and mass parameters and conditions of its installation on a concrete object.

The length $L_{1\text{-}2}$ of the flat inclined part of a ramp is connected with the height ($H_{1\text{-}2}$) which is proved by the following dependence:

$$L_{1-2} = \frac{H_{1-2}}{\sin\theta_{ramp}}. \qquad (4)$$

While moving along a ramp, a UAV is affected by the gravity component $G\sin\theta_{ramp}$, which results in a decrease in acceleration as seen in the following formula: $\Delta j_x = -g\sin\theta_{ramp}$, consequently, the general acceleration equals the value that is calculated as follows: $j_x = j_{cp} - g\sin\theta_{ramp}$. Taking into account that $g\sin\theta_{mp}$ is a constant value, the general movement remains accelerated at regular intervals and variation of speed is equal to the value that is calculated as follows:

$$V_2^2 - V_1^2 = 2j_x L_{1-2} = \\ 2j_{cp}L_{1-2} - 2gL_{1-2}\sin\theta_{ramp} = 2j_{cp}L_{1-2} - 2gH_{ramp}. \qquad (5)$$

Value $2gL_{1\text{-}2}sin\theta_{ramp}$ is a complex one that calculates a decrease in speed taking into account the climb angle of a ramp. The resulted formulas show that the value of UAV acceleration during the running start is determined, basically, by its thrust-to-weight ratio and the climb angle of a ramp. The greater is the acceleration a UAV moves with, the shorter the part of a runway that it has for gaining the speed required for a launch (Fig. 3).

If a ramp has a cylindrical surface (Fig. 2), its climb angle varies from 0 to θ_{ramp}. Braking force $Gsin\theta$ will gradually increase from 0 at the beginning of a ramp to $Gsin\theta_{ramp}$ at the end of a ramp. As force $Gsin\theta$ influences the variations in kinetic energy by means of losing height, the loss of speed, due to the effect of this force,

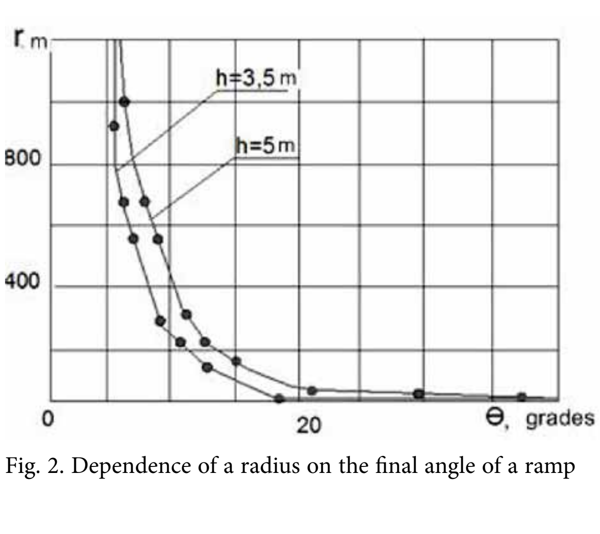

Fig. 2. Dependence of a radius on the final angle of a ramp

will not depend on the ramp form, and is determined only by height and calculated in accordance with formula (5).

It can be assumed that the curvilinear part of a ramp is a cylindrical surface of a constant radius, and its height, for constructive reasons, is limited by value H_{mp}. Evidently, figure 4 shows that $AB = r\cos\theta_{ramp}$, $B_1 = r$, $H_{ramp} = r - r\cos\theta_{ramp}$, $B_1 = r$. Consequently, it is possible to find a connection between radius (r), height (H_{ramp}) and the final angle of a ramp:

$$r = \frac{H_{ramp}}{1 - \cos\theta_{ramp}}. \qquad (6)$$

This formula shows that the greater the angle θ_{ramp} is, the less radius a ramp should have at the specified height.

Figure 4 illustrates the dependence of r on the final angle of rotation of the surfaces of the ramps with the heights $H_{ramp} = 3.5$ meters and $H_{ramp} = 5$ meters. The above mentioned figure (Fig. 4) leads us to conclude that the strongest influence of value θ_{ramp} on value r is observed at small angles of θ_{ramp}. To increase angle θ_{ramp} from 7° to 20°, the radius of a ramp ought to be reduced from 600 m to 90 m, i.e. almost seven times. However, while reducing value r, the centripetal force affecting a UAV increases and normal reload (n_{ya}) increases as well.

As speeds of a UAV's movement are insignificant, reload is made basically by the normal reactions of a ramp $N = N_1 + N_2$, which is applied to the gear of a UAV (Fig. 1). Depending on the speed of a UAV's movement during the launch, the length of the horizontal part of a ramp can be calculated using formula (2). The dependence of the length of the horizontal part of a ramp on the speed of a UAV's movement is shown in figure 5.

It is known that the curvilinear character of a UAV's movement is determined by centrifugal force $\frac{mV^2}{r}$ and centripetal force $Y_a + P_y + N - G\cos\theta$. Their equality makes it possible to obtain an expression for the radius of the curvature of a trajectory:

$$r = \frac{GV^2}{g(Y_a + P_y + N - G\cdot\cos\theta)} = \frac{V^2}{n_y - \cos\theta}, \qquad (7)$$

where $n_{ya} = \dfrac{Y_a + P_y + N}{G}$.

The diagram in figure 3 shows the dependence of normal reload n_{ya} on the radius of the curvature of a ramp. It is evident that normal reload starts to increase intensively under the reduction of a radius to less than 100 meters. So, if a UAV moves on a curvilinear surface with a 100 m radius, it will be affected by a normal overload n_{ya} equal to 4. If value r is equal to 50 meters at the same speed, a UAV will be affected by an overload equal to 7.

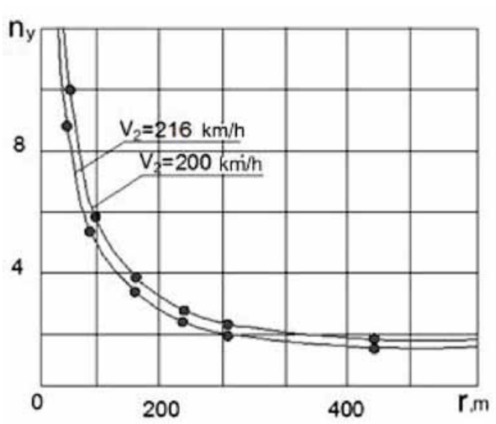

Fig. 3. Dependence of an overload on the radius of a ramp

It is natural that the operation of a UAV with such a reload can seem to be extremely difficult after a launch from a ramp. Formula (7) shows that the overload increases approximately to the square of speed at constant radius; hence, a UAV will experience the greatest overloads at the end of a ramp. Therefore, to reduce loading on a UAV frame, it would be useful to construct a ramp having a variable radius, in particular, with a small radius in the beginning and increasing in the final part. While moving on a curvilinear surface, the angular speed of a UAV turn trajectory is determined by the linear speed and the radius of the curvature:

$$\frac{d\theta}{dt} = \frac{V}{r}. \tag{8}$$

The distance that a UAV goes through on the curvilinear part of a ramp can be defined as the length of an arc, which is calculated as 1–2 (Fig. 4):

$$L_{1-2} = r \cdot \theta. \tag{9}$$

The obtained expressions allow developing an algorithm and the methodology for calculating the characteristics of a launch of a UAV belonging to the aircraft family from a ramp.

Using this technique, it is possible to carry out the calculation of the required parameters of a ramp for different aircraft types. One variant of the calculation procedure is provided below:

- To specify the height of a ramp as H_{ramp} (for constructive reasons) and the speed while lifting off from a ramp V_2 (for safety).
- To calculate the final climb angle of a ramp from the conditions for obtaining optimal trajectory.
- Use formulas (4) or (9), to calculate the length of a flat or curvilinear part of a ramp.
- Use formula (5), to determine speed V_1, and to calculate the length of a horizontal part of a ramp (L_{0-1}), using formula (2).

Taking into account constant changes in atmospheric conditions, and the requirements for the fulfillment of flight tasks which can lead to changes in weight and size, operational and other properties, for different types of UAVs, this methodology can be implemented into the Decision Support System (DSS).

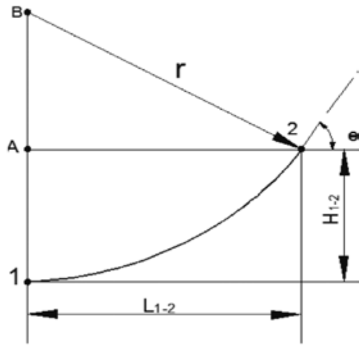

Fig. 4. The diagram of the curvilinear part of a ramp

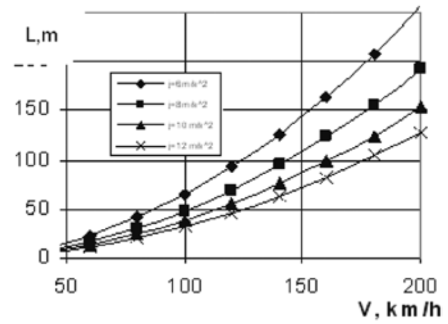

Fig. 5. Dependence of the length of the horizontal part on the speed of movement

Having considered the characteristics of a ramp, characteristics of a UAV and flight missions, it can be presumed that the application of the Decision Support System helps to provide real time recommendations concerning the variations of a UAV payload and the specific features for controlling a launch from a ramp.

From the formulas given above, it is clear that a UAV gains some speed and angle of climb, when lifting off from a ramp. The subsequent movement of a UAV depends on these parameters. The next step is to consider what requirements are relevant to the parameters of UAV movement while lifting off from a ramp in order to make the subsequent trajectory optimal.

3. Optimization of the trajectory of UAV lift -off from a launcher

Small and medium UAVs usually liftoff with the help of a launcher (launching device), which is defined as a platform inclined to the horizon at angle θ_2 or as a ramp (Iliushko *et al.* 2009). A launcher is installed on a vehicle in order to increase mobility. To improve acceleration while lifting off, a launcher is equipped with an additional booster that provides specified speed V_2 of a lift-off from a platform. It is evident that the subsequent movement of a UAV will depend on both of these parameters.

A powerful launching device allows accelerating a UAV to the speed of lift-off and more, analogically to an average takeoff. However, in this case, a launcher would be weighty and consume a lot of energy, and, consequently, expensive and inconvenient to use. Therefore, while lifting off from a launcher, the speed of a UAV's movement is specified to be as low as possible (less than the usual speed of a lift-off), and the launcher itself is installed at a certain angle in order to make a UAV move along a half-ballistic trajectory in the beginning of the flight, increasing the speed upon leaving the platform of a launcher.

It is also obvious that the angle of the installation of a launcher ought to be an optimum value. If this value is considerably exceeded, a UAV can climb intensively, but lose speed up to the stall. If angles θ_2 are small, a UAV gains speed more intensively, but loses altitude and can hit some objects on the ground.

Thus, the parameters of a UAV flight trajectory while passing the airborne part and the characteristics of a launcher appear to be closely related.

Due to the optimization of the trajectory (selecting optimal values θ_2 and V_2), an effective launching device can be obtained according to its geometric and mass parameters. The optimal trajectory can be found using the methods of variational calculation (Kiselev *et al.* 2008). To do this, the equations defining the trajectory of a UAV's vertical movement should be formulated as follows:

$$\begin{aligned}
&\frac{dV}{dt} = g(n_{xa} - \sin\theta), \quad n_{xa} = \frac{P + R_x - X_a - F}{gm};\\
&X_a = C_{xa}\frac{\rho V^2}{2}S;\\
&\frac{d\theta}{dt} = \frac{g}{V}(n_{ya} - \cos\theta), \quad n_{ya} = \frac{Y_a + R_y + N}{gm};\\
&Y_a = C_{ya}\frac{\rho V^2}{2}S;\\
&\frac{dH}{dt} = V_y = V\sin\theta; \quad \frac{dL}{dt} = V_x = V\cos\theta;\\
&\frac{dH_e}{dt} = V_y^{*} = Vn_{xa}; \quad H_e = H + \frac{V^2}{2g},
\end{aligned} \tag{10}$$

where V and H are speed and height (altitude), θ – climb angle of the trajectory; n_{xa} and n_{ya} – tangential overload and normal overload; H and H_e – geometric energetic components of height; P – sustaining power; R_x and R_y – tangential and normal components of the traction of a booster; X_a and Y_a – drag and lift; F and N – normal and tangential components of the reaction of pillars. After a UAV's lift-off from a launcher, it can be described as $R_x = R_y = F = N = 0$.

The most critical parameter of a launch is height, so the optimization is developed on the basis of the minimum height loss (Silkov 1997). It can be assumed that the elevator is pre-installed in the balancing position corresponding to the maximum allowable angle of attack, so after losing contact between a launcher and a UAV, it remains balanced time after time. Consequently, the subsequent flight will be performed along a half-ballistic trajectory under the condition that the maximum coefficient of lifting force remains stable. The form of the trajectory will significantly depend on the climb angle θ_2 of a launcher.

During the flight, speed and height change simultaneously; therefore, both of these parameters should be included into the independent variable component. This component is determined as energetic height H_e that depends on the geometric height and the square of speed and determines the total energy of 1 kg of a UAV's mass. In this approach, the variations of geometric height ΔH pertaining to the starting position (Fig. 1, point 2) ought to be calculated as the functional of energetic height according to the following formula:

$$\Delta H_{\min} = \int_{He1}^{He2} \frac{dH}{dH_e}(V,\theta,n_{xa},n_{ya})dH_e \longrightarrow \min. \tag{11}$$

The derivative dH/dH_e can be calculated by means of dividing the third equation by the fourth one of equation system (10) according to the following formula:

$$\frac{dH}{dH_e} = \frac{\sin\theta}{n_{xa}}. \tag{12}$$

The resulting ratio has a certain physical meaning, in particular, due to $\sin\theta$, the influence of gravitational force $G \sin\theta$ is evaluated, and owing to n_{xa}, the influence of traction force on variations in height is evaluated. $\sin\theta$ and n_{xa} can be determined as a function of time by solving the system of equations in (10). At the same time dependence $H_e(t)$ can be calculated, and then ratio $\sin\theta / n_{xa}$ regarding function H_e can be calculated as well.

To illustrate these claims, a table 1 for a UAV with the following characteristics has been developed:

Table 1. Initial data for calculation

t_0 s	Δt s	V_2 км/h	H_2 m	θ_2 grad	m кg	C_{ya}	C_{xa}	P kg/m^3	S m^2	P H
2.00	0.5	95	2.29	4	149	1.2	0.08	1.225	2.15	350

The limits of integration are determined by the initial values of height and speed (the moment a UAV leaves the platform of a launcher) and the final values of height and speed (the final point of the trajectory of a lift-off), which is seen in the following formulas: $H_{e2} = H_2 + V_2^2/2g$ and $H_{e4} = H_4 + V_4^2/2g$. It should be noted that value H_{e21} is determined according to the data of a launcher, and value H_{e4} is determined according to the conditions required for resuming a safe flight after lift-off. So, for piloted aircraft, value H_4 = 10.7 m, and value $V_4 = 1.2\ V_s$, where V_s is stall speed.

Table 2. Variation of flight parameters in a time dimension

t, s	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
V km/h	95	95	95	96	98	101	104	108	111	115	119	122	125
θ grad	10	10	7	5	2	0	−1	−2	−3	−2	−1	0	2
H m	2.29	5.03	7.22	8.85	9.91	10.44	10.48	10.15	9.56	8.86	8.23	7.84	7.90
Xa H	73	73	73	75	78	83	88	94	101	108	115	121	126
Ya H	1100	1093	1102	1129	1175	1238	1317	1409	1511	1616	1719	1815	1896
n_{xa}	0.19	0.19	0.19	0.19	0.19	0.18	0.18	0.18	0.17	0.17	0.16	0.16	0.15
n_{ya}	0.75	0.75	0.75	0.77	0.80	0.85	0.90	0.97	1.03	1.11	1.18	1.24	1.30
He m	37.78	40.28	42.78	45.28	47.81	50.37	52.97	55.61	58.28	60.97	63.67	66.37	69.06
(Sinθ)/ n_{xa}	1.10	0.88	0.65	0.42	0.21	0.02	−0.13	−0.23	−0.27	−0.24	−0.14	0.02	0.25

For these conditions, and in accordance with the system of equations in (10), the basic parameters of the trajectory of a UAV movement (Tab. 2) can be calculated relative to time for an angle θ_2 equal to 10°.

The last two rows of the table allow getting the desired dependence $dH/dH_e = f(H_e)$ in accordance with formula (12) and determine the functional in formula (11).

According to similar tables, dH/dH_e is calculated for a few more climb angles θ_2 of a launcher and their graphical curves are constructed (Fig. 6). Figure 6 illustrates that when the angle of the installation of a launcher increases from 2 (to 15), the calculated curves shift to the right, climb up and then go down. At least one of the curves appears to be the lowest one by its absolute value. For the given initial conditions, this curve corresponds to angle θ_2 = 10. This angle is optimal and ensures the minimum loss of height.

The resulting curve is used to calculate the quantitative value of the loss of height. This curve is shown in figure 7 and is integrated by value H_e. As a result, the variation of geometric height can be obtained pertaining to the point where a UAV leaves the platform of a launcher (point 2).

This figure also shows that a UAV starts climbing after leaving the platform of a launcher. Taking into consideration that lifting force is weaker than gravitational force, the climb angle of the trajectory begins to decrease and reaches zero at point 2, so a UAV gains the maximum height. Afterwards, derivative dH/dH_e is lower than zero and the stage of descent begins, the speed increases, lifting force increases, so a UAV begins to level. At point 3, the climb angle of the trajectory is equal to zero, and the height is minimum (−3.4 m under the condition that value H_e = 58.9 m). After passing point 3, a UAV begins to climb and can be transferred to controlled flight if safe speed and safe height are achieved.

It should be noted that the loss of height occurs when the values of derivative dH/dH_e are negative. This loss is quantitatively determined by the square between this function and the y-axis (Fig. 7, s). When the values of derivative dH/dH_e are positive, a UAV climbs.

Thus, if an interval of integration is given as (H_{e1} – H_{e2}), the maximum loss of height is certainly related to the extreme value of derivative dH/dH_e. Therefore, after formulating the dependence of the minimum values of the derivative on energetic height under the condition that there are various values of the climb angles of a launcher, the optimal climb angle of this launcher can be determined, as shown in figure 8.

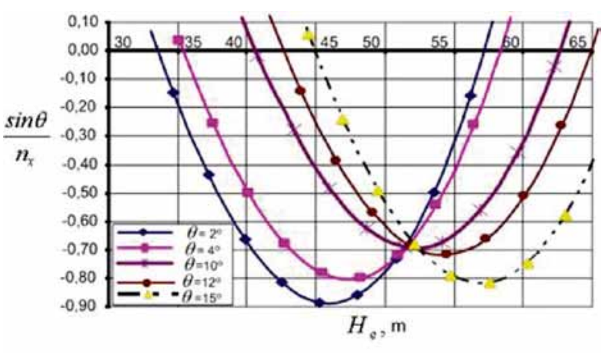

Fig. 6. Dependence of derivative dH/dH_e on value H_e for various angles of the installation of a launcher under the condition that V = 95 km/h, P = 350 N

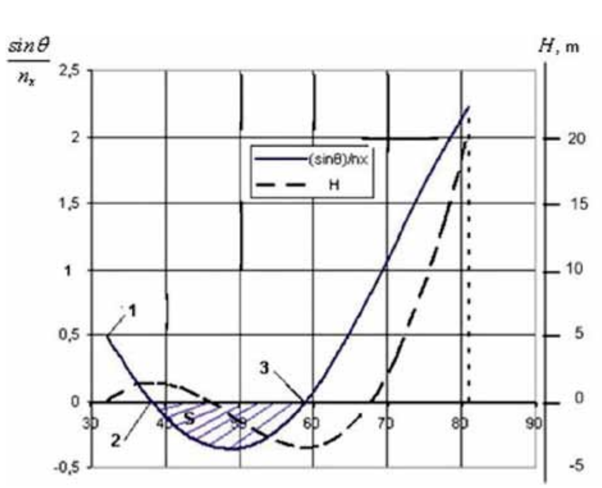

Fig. 7. Determination of the minimum height of descent under the conditions that P = 350 N, θ_2 = 10°

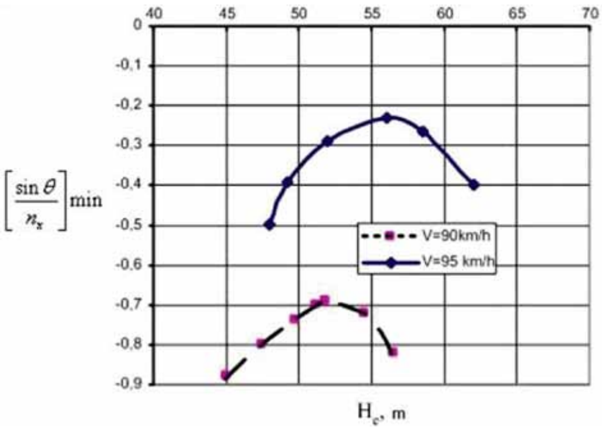

Fig. 8. Dependence of the minimum values of derivative dH/dH_e on H_e value under the condition that P = 350 N

However, the alteration of height is influenced not only by the climb angle of a launcher, but also significantly by initial flight speed V_2. Obviously, if this speed is greater than the speed of a lift-off, the loss of height will not take place anyway and a UAV can immediately perform a controlled flight. As previously noted, in this case, a launcher has negative characteristics adversely affecting its operation and cost. Therefore, after determining the optimal climb angle of a launcher, the optimal speed of a UAV launch ought to be determined. Taking this into consideration, it should be kept in mind that the optimal angle θ_2 does not vary substantially if speed is not changed significantly.

In this case, it is necessary to construct two or three curves $dH/dH_e = f(H_e)$ for different speeds in accordance with the described method, as shown in figure 8, and determine a minimum height value at each point for these curves. Then, some height values depending on the speeds of a launch can be obtained, the minimum safe height can be determined, and the desired speed of a UAV lift-off from a launcher can be selected with the help of the interpolation method. The booster power can be determined by the required value of the speed of a lift-off. The proposed method is approximate because it does not take into account the transient changes in the angle of attack after losing contact with a launcher. The above mentioned results should be considered with respect to the time not from the moment a UAV leaves the platform of a launching device, but from the end of the transition process pertaining to the angle of attack. This distance can be calculated independently, and the resulting parameters can be added to the initial ones. By gathering the resulting parameters of a UAV's launch, its geometric data can be calculated, as well as the power of a booster. Together, they are supposed to provide the required characteristics of a UAV's lift-off from a launching device.

4. Conclusions

1. The advantages of a UAV launch from a ramp have been shown.
2. The kinematic parameters of movement during a UAV launch from a ramp have been analyzed.
3. The required formulas have been calculated to determine the kinematic parameters of movement during a UAV launch in compliance with the specified geometric data of a ramp.
4. The method of calculating rational geometrical parameters of a ramp has been developed. These parameters provide a safe lift-off of a UAV with the specified flight data.
5. The algorithm for determining the optimal trajectory of a UAV launch has been developed according to the variational method. A numerical example of using this algorithm has been provided.

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