ISSN 1648-7788 / eISSN 1822-4180

VILNIUS TECH Vilnius Gediminas Technical University

AVIATION

2025
Volume 29
Issue 3
Pages 137–149
https://doi.org/10.3846/aviation.2025.24456

MODAL AND RANDOM VIBRATION ANALYSIS FOR STRUCTURAL DESIGN OF AIRCRAFT EXTERNAL STORES

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Article History:
- received 7 February 2025
- accepted 4 June 2025

Abstract. This study introduces a novel methodology for the structural design of aircraft external stores, focusing on the integration of modal and random vibration analysis through the Power Spectral Density (PSD) method. The primary goal is to develop a robust design approach that enhances structural reliability and avoids resonance, a critical challenge in aerospace applications. Unlike conventional methods, this research presents a convergent strategy combining Finite Element Analysis (FEA), Computational Fluid Dynamics (CFD), and fluid-structure interaction to accurately predict stress, displacement, and frequency responses under dynamic loading. The methodology is specifically applied to external stores mounted at the fuselage station of a subsonic turboprop aircraft. A key novelty of the work is the validation of structural response through PSD-based vibration analysis, offering a high-fidelity prediction of dynamic behavior. Results indicate a close correlation between FEA and vibration outcomes, with maximum stress and deformation well within material limits and outside the aircraft's natural frequency range, thus avoiding resonance effects. The conclusions of this study provide a roadmap for the design of aircraft stores based on modal and random vibration analysis. The proposed methodology helps to understand aeroelastic behavior, avoid resonance, resist aerodynamic forces, and prevent structural damage to aircraft external stores.

Keywords: aircraft external store, structural design, aeroelasticity, random vibration, power spectral density, fluid-structure interaction.

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1. Introduction

Modern advancements in warfare, navigation, and communications have been made possible by consistent breakthroughs in aerospace devices, electronics, and integrated circuits (Velastegui et al., 2022). Aircraft fulfill multipurpose missions using external stores, which are devices used by airplanes to transport weapons, fuel, or electronic devices to increase their payload, improve their performance, or implement unique systems designed to fulfill specific missions. Nowadays, modular and reconfigurable aircraft use this technology, generating much attention for use in aerospace and military systems. This multipurpose configuration allows the change of the mission profile by the interchange of some components, elements, or stores, increasing the system's functionality (Sahoo et al., 2020). This approach will boost purpose capability and flexibility across necessary missions, especially for military applications. Some authors have noted the tendency toward fewer aircraft types and increased multimission capability simultaneously (Mena-Arciniega et al., 2024)

It is improbable that specialized platforms could be acquired for every kind of mission because the cost of designing a single-purpose aircraft has risen to an unaffordable level. Aircraft designs must be adaptable and customizable to complete various missions to fulfill all future needs (Criollo et al., 2024). Therefore, advanced aircraft are frequently built with modular airframes and can easily hold external stores to interchange advanced sensors and weapons.

External stores are increasingly being used to support the flexibility and interoperability of aircraft. Compact technological systems allow the use of external stores for a variety of tasks, such as radar imaging, signal intelligence, or electro-optical images. By switching out the external store connected to the aircraft, a plane outfitted with an external store for one purpose, such as recognition, can be rapidly modified to serve another mission, like radar imaging.

In the digital twin era, reliable design approaches are highly prized in the aeronautical industry because they ensure safe operations, minimize structural weight and resources, and maintain high-quality products. Therefore, alternative engineering models incorporate different variables than those typically utilized to compare traditional design methods.

The research questions are related to understanding the correlation between the structural design of external PODs based on vibration outcomes and determining how modal and random vibration analysis converges with other numerical and experimental approaches. The study uses a Power Spectral Density (PSD) method to evaluate the design approach.

Designing an aircraft's external store involves many approaches, including dynamic and static response and an aeroelasticity study of the device. The structural integrity of this component is essential for the aircraft to operate safely within the parameters of airworthiness and safe flight, so its design and production must be ensured with an adequate safety factor (Criollo et al., 2021). Therefore, the redundancy of design methods presents a solution to consider for validating the results of various studies.

One of the most significant challenges in aircraft structural design is the consideration of aeroelastic effects. Kim Dong-Hyun presented a cutting-edge aeroelastic analysis for an aircraft with external stores using computational structural dynamics (CSD). Their study uses Euler equations to account for complex structure configurations; this computational approach is based on the model-based coupled nonlinear analysis with the matched-point concept, which can provide highly accurate and valuable engineering data on the structural design of advanced flight vehicles (Kim et al., 2003).

Numerous studies have explored the aeroelastic behavior of aircraft with external stores, often emphasizing aerodynamic or structural dynamics using classical or computational methods. For instance, Kim et al. (2003) utilized computational structural dynamics (CSD) to evaluate flutter in full aircraft configurations with pylons and stores. Similarly, Melville (2001) conducted a nonlinear simulation of an F-16 using Navier–Stokes solvers, providing detailed insights into global aircraft dynamics, but lacking integration with vibration-based structural design.

Pollock et al. (2012) employed the doublet lattice method to predict wing/store flutter across various flight regimes, while Padmanabhan et al. (2016) examined mass and stiffness variations due to external stores on F-18 wings. Though effective for aerodynamic modeling, these approaches did not consider random vibration or power spectral density (PSD) effects, which are significant under operational turbulence and engine-induced vibrations.

Tamer (2021) applied flexible multibody dynamics to show how mass location influences natural frequencies, whereas Kim et al. (2003) focused on reducing structural weight by refining aeroelastic boundary predictions. These methods primarily addressed deterministic responses.

In contrast, PSD-based approaches, such as those by Reytier et al. (2012), modeled fatigue under gust loads using stochastic PSD inputs. Bonte et al. (2007) developed PSD-based stress models for fatigue, and Youngworth et al. (2005) outlined PSD theory in optics and mechanics. However, these works rarely addressed full-system integration or design validation through both modal and random vibration analysis. Therefore, the present study contributes a novel and integrated methodology, combining CFD, FEA, modal, and PSD-based vibration analysis for the structural design of external stores. This approach directly addresses the gap between deterministic simulation techniques and the need for dynamic, stochastic validation in operational aerospace environments.

The state-of-the-art focuses on the aerodynamic and aeroelastic effects of external stores on the aircraft's wing by using several methods, numerical techniques, and computational tools to demonstrate how fluid-structure interaction affects structural design. All this analysis has been performed using theoretical approaches such as Euler equations, Navier Stokes solvers, Lattice methods, or Hamilton's variational principle. Although several estimations have been studied in detail, this study focuses its attention to modal and random vibration analysis using (PSD) approximation to estimate a structural design method for an aircraft's external stores. The present research aims to innovate in structural design and aeroelasticity by trying to identify a convergence method for designing this kind of aeronautical component.

The purpose of this paper is to evaluate the convergence of the methodology for designing aircraft external stores based on modal and random vibration analysis using data from computational simulation models built using Computational Fluid Dynamics (CFD) and Finite Element Analysis (FEA) without altering the aircraft configuration or requiring significant certification, a novelty result that can optimize the design process.

The design approach bases its decision on the stress and strain generated by the boundary conditions, ensuring that the loads transferred to the structure are adequate for the material capacities. The design must sustain the structure robustly enough to tolerate these stresses, considering an optimal safety factor and the material's elastic range. The methodology aims to produce displacements and stresses close enough to validate good convergence.

The present research investigates a method for the structural design of an aircraft's external store based on modal and random vibration analysis for a subsonic turboprop aircraft in a configuration that permits the transportation of electronic equipment from the ventral station of the aircraft's fuselage. Results are compared and analyzed respectively to determine the level of convergence and validation. These are evaluated under safe flight operating conditions, mechanical response, and aeroelasticity effects. The level of convergence, applications, and validation of results are also discussed.

2. Methodology

The methodology begins by defining the geometry of the external store structure using computer-aided design

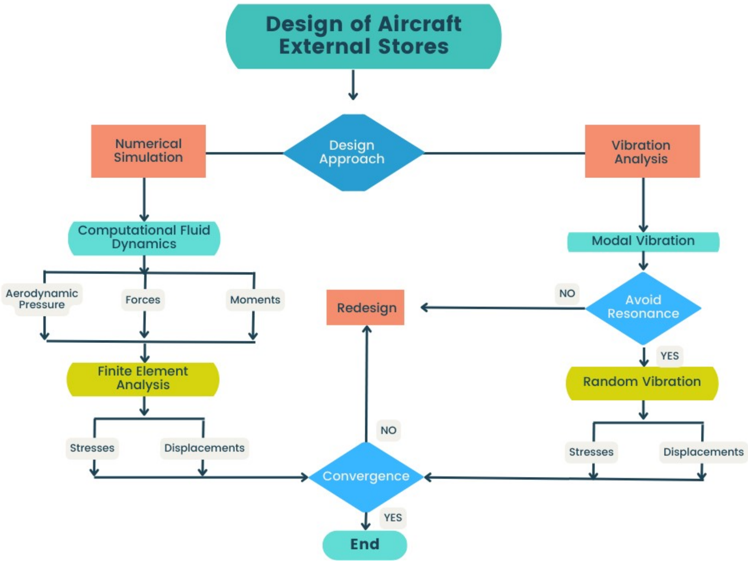

Figure 1. Technology roadmap

(CAD) methods. Next, critical flight load conditions are applied to the structure using CFD. Once the pressures, moments, and forces to which the structure are determined, FEA is used to identify the stresses and deformations that the structure will suffer in critical conditions under an adequate safety factor. This FEA model is compared with the computational design from the perspective of vibration analysis under the criteria defined in the military standard Environmental Engineering Considerations and Laboratory Tests - Vibrations MIL-STD-810E, which allows for the determination of the external store's natural frequencies and the study of random vibrations under the statistical approximation of the PSD function (U.S. Army Test and Evaluation Command, 2020).

Figure 1 comprehensively outlines the methodology for the structural design of aircraft external stores, integrating both numerical simulation and vibration analysis. The process begins with the initial design of aircraft external stores, which then branches into two main analytical approaches. On one hand, numerical simulation involves Computational Fluid Dynamics (CFD) to determine aerodynamic pressures, forces, and moments acting on the device. These outputs are subsequently fed into Finite Element Analysis (FEA) to calculate stresses and displacements. Simultaneously, the vibration analysis branch delves into Modal Vibration to identify the natural frequencies and modes, which are crucial for assessing and ensuring the design can avoid resonance. If resonance cannot be sufficiently mitigated, a redesign is initiated; otherwise, the analysis proceeds to Random Vibration to determine stresses and displacements under realistic dynamic conditions. Both the FEA and random vibration analysis results converge at a decision point. If the design's structural performance (in terms of stresses and displacements) is deemed unacceptable, or if resonance remains a concern, the process loops back to a redesign. This iterative cycle of design, analysis, and refinement continues until all structural integrity and vibration criteria are met, leading to the end of the design process, signifying a validated and robust structural design for the aircraft external stores.

The path to follow in the first instance is the definition of the geometry of the external store structure, and simulate a design method to define the critical flight load conditions to be applied to the structure, using the CFD method, taking into account the provisions of directive Acceptable Methods, Techniques, and Practices – Aircraft Alterations AC 43.13.2B (Federal Aviation Administration, 2008), and the characteristics established in the flight manual of the aircraft under study (Embraer, 2012c).

Once the pressures, moments, and forces to which the structure will be subjected have been determined, it is proceeded to the finite element analysis. This permits the identification of stresses and deformations the structure will suffer in critical conditions under an adequate load factor.

For the comparison between the methods for the design of aircraft external stores with the finite element analysis, the analysis of modal and random vibrations is used, whose evaluation foundations are chosen following the provisions of the MIL-STD-810F standard. This regulation introduces the existing theories on vibration analysis; the terminology, methodology, requirements, failure criteria, and descriptions are taken into this standard, whose procedures are based on international regulations to determine the vibration exposure of a life cycle and allow the determination of the main natural frequencies of the external store structure with the study of random vibrations under the statistical approximation of the PSD function.

2.1. Structural model and layout

From a structural perspective, the design of an external store must consider the following factors: geometry, location, accessibility, airframe modifications, aircraft performance, and specific requirements of the electronic equipment. Figure 2 shows the detailed configuration of the external storage structure designed for fuselage mounting on a subsonic turboprop aircraft. The structure follows a semi-monocoque design philosophy, which combines load-bearing skin with internal reinforcements to balance weight and strength.

The main body of the structure is constructed from AISI 304 stainless steel, providing high strength, corrosion resistance, and structural rigidity. It contains the primary load-bearing elements, including spars, ribs, and stringers, arranged longitudinally and transversely to resist bending and torsional loads during flight. The stainless-steel core also serves as the mounting base for the lug suspension system, which connects the store securely to the fuselage pylon using standard aerospace hardpoints.

The head and tail sections are fabricated from carbon fiber woven fabric prepreg, chosen for its lightweight properties and excellent stiffness-to-weight ratio. These sections are also reinforced internally with composite ribs and stringers, ensuring structural continuity and aerodynamic integrity. The lifting surface, also composed of carbon fiber, contributes minor aerodynamic lift while aiding in dynamic stability.

The lug suspension system is positioned above the central core and serves as the primary load transfer interface between the external store and the aircraft. It is designed to comply with MIL-STD and FAA installation standards, supporting quick integration or removal without affecting aircraft certification status. The entire structure encloses a cavity suitable for electronic payloads, sensors, or mission-specific hardware, shielded from environmental and vibrational loads.

During this study, the fuel external tank P/N 314-19169-401 of the Embraer 314 aircraft is used as the basis for the external geometry selection (Embraer, 2012a). The prototype has the same profile and aerodynamic characteristics as the aircraft fuel tank, which leads to the assumption that there will be no changes in the weight and balance of the aircraft and no modifications to performance.

Military aircraft have hard points beneath their fuselage and wings, initially designed for carrying weapons and fuel tanks. These hard points can also be used to carry external stores. However, wing-mounted pylons can affect the aircraft's performance and are less suitable for wiring runs needed for electronic devices (Alksninis & Fisher, 2009). The ideal way to structurally attach an external store to a fuselage is to use a pylon or hardpoint with a built-in standard adapter unit to install and remove the pod and airframe interoperability easily. On the condition that ground clearance is accessible, fuselage hardpoints give the airframe more interface freedom (Haro et al., 2023). Fuselage hardpoints are generally more straightforward to connect and support than wing pylons. External sheets at fuselage stations inside or close to the hardpoint provide access for electrical routing from the primary system to the electronic devices. Therefore, this research aims to define a methodology for designing an external store mounted on a fuselage pylon station with direct wiring facilities from the aircraft's electrical system.

The prototype pod is fastened to the aircraft using the Lug Suspension component (Department of Defense, 2008), which is used to secure the aircraft's external store at the different fuselage stations. Pylon airframe modifi-

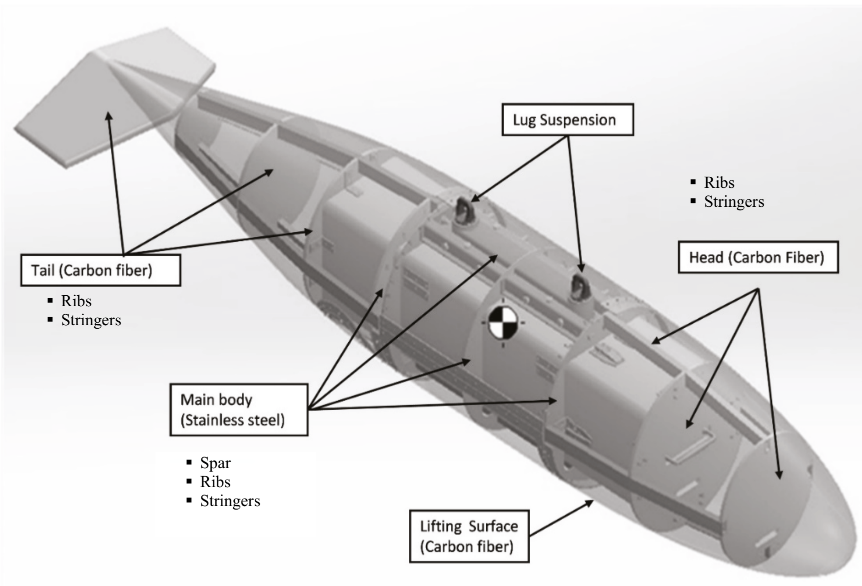

Figure 2. External storage structure and setup

cations can be installed temporarily and removed when necessary, allowing the airframe to maintain its standard certification and operational status when the pod and pylon are removed (Federal Aviation Administration, 2007).

Materials for aircraft structures must provide sufficient strength and robustness to sustain in-service loads, as well as corrosion and high temperatures from the environment and internal devices. The prototype structure weighs 61.32 kilograms, based on the application of materials, geometries, and densities used in its construction. A 3 mm thick stainless steel 304 sheet is used for the main beam, which exhibits isotropic behavior, with a density of 7750 kg/m^3 and a tensile ultimate yield strength of 207 MPa. The lug suspension components are attached to the main beam, forming a structural box to house the electronic equipment.

Alternatively, high-strength carbon fiber woven fabric prepreg is used for the ribs and stringers of the head and tail of the structure, as well as for the entire coating. The composite material has a 1575 kg/m^3 density and a tensile yield strength of 430 MPa. The configuration is a quasi-isotropic lamination of 0/90/45/-45/-45/45/90/0, with a thickness of 2.5 mm to respond identically regardless of the direction of loading. The prototype has a span of 2352 mm and a diameter of 390 mm.

2.2. Aerodynamic configuration

Boundary conditions consider different aerodynamic effects caused by the POD implementation to estimate the maximum aerodynamic load on the prototype structure in the straight and level flight stages at the maximum structural speed (V_{no}) and in the takeoff and landing stages following the Aircraft Flight Manual. For each flight phase (takeoff, landing, cruise), it is essential to note that the boundary conditions vary, affecting the values of velocity, angle of attack, altitude, temperature, density, pressure, and viscosity. These values are approximated based on International Standard Atmosphere (ISA) calculations (Cavcar, 2000). The CFD simulation is performed under a subsonic turbulent regime. Critical flight conditions were estimated to determine the most critical pressures, forces, and moments the structure must support. Table 1 shows the boundary conditions applied.

2.3. Static model and load analysis

For the structural analysis, the loads applied to the structural core of the external store, whose effect of transmission of forces and moments will be transferred from its coating to the pylon of the fuselage station, have been considered. The effects generated by the forces, pressures, and moments applied to the structure are simulated by finite element analysis to determine where the most critical stresses and displacements occur. Aircraft structures are exposed to various loads in flight, and the goal is to predict the stresses and displacements that the design may experience and to construct the structure robustly enough to tolerate them. Once the aerodynamic loads are estimated, the total pressures, forces, and moments the external store structure should withstand must be determined to calculate the maximum structural load requirement, under the most critical aircraft operating conditions (Federal Aviation Administration, 2016). According to the structural data of the Advisor Circular "Acceptable methods, techniques, and practices for aircraft alterations – AC 43.13-2B" (Federal Aviation Administration, 2008), the limit load factors must be assessed, which are the maximum load factors that may be expected during service (maneuvers, gusts, or ground load factors) with an adequate safety factor.

Aircraft load analysis was performed using the procedure established in AC 43.13-2B 2008). This technique outlines the steps for adequate structural calculation, beginning with determining loads, material, and geometry, then calculating stresses, and ending with comparing the material properties to the estimated safety factor. The highest load values from the CFD simulation were considered for simulation and structural analysis to ensure the integrity of the prototype structure and its interaction with the aircraft.

The loads used for the structural simulation were determined by multiplying the mass values by their load factor according to their direction (sideward, upward, forward, downward, rearward) to comply with the limit load factors for acceptable equipment installations. The fixed supports are established in the Lug suspension components where the POD joins to the fuselage pylon station. Distributed pressure and aerodynamic forces are imported from CFD simulation. Table 2 shows the structural simulation and analysis's maximum aerodynamic forces and moments. A conservative design approach was used to ensure

Table 1. Aerodynamic boundary conditions

Phase of flight	Speed [m/s]	Angle of attack [°]	Temperature [°C]	Density [kg/m^3]	Static Pressure [Pa]	Cinematic Viscosity [m^2/s]	Dynamic Viscosity [kg/m·s]
Takeoff	82.31	7.5	15	1.225	101325	1.470e-5	1.802e-5
Landing	56.59	–3	15	1.225	101325	1.470e-5	1.802e-5
Cruise at 1500 [m]	144	0	5.1	1.056	84307	1.373e-5	1.741e-5
Cruise at 3000 [m]	144	0	–4.8	0.904	69682	1.291e-5	1.692e-5
Cruise at 4500 [m]	144	0	–14.7	0.771	57182	1.203e-5	1.642e-5

Table 2. Limit load factors

Direction	Load Factor	Applied Force
Sideward	1.5 g	902.32 [N]
Upward	4.5 g	2706.97 [N]
Forward	9.0 g	6002.61 [N]
Downward	9.0 g	5413.94 [N]
Moment (Z)	------	153.48 [Nm]

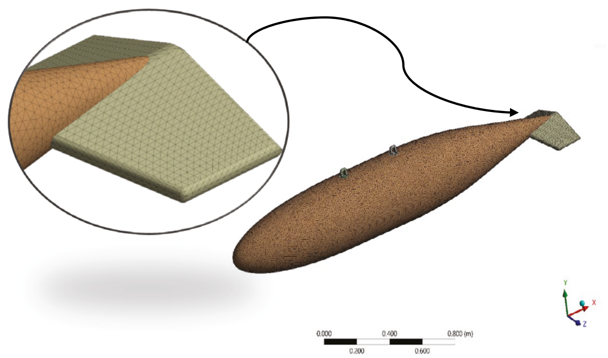

Figure 3. Mesh sizing and control for FEA

the integrity of the prototype structure and its interaction with the aircraft.

The mesh used for the structural simulation is shown in Figure 3 where the finite element model consists of tetrahedral solid elements, suitable for capturing high-gradient stress regions in complex geometries. Mesh quality was ensured using face, body, and volume mesh controls, with an element size of 1 mm. The mesh comprised approximately 1.4 million elements and 245,000 nodes, maintaining an orthogonal quality greater than 90% and an aspect ratio close to 1. This guarantees high fidelity in both the structural and dynamic response calculations. The simulations were carried out using ANSYS Workbench 2022 R2, which integrates preprocessing, solver execution, and postprocessing in a unified platform. This software was chosen for its advanced meshing capabilities, built-in random vibration analysis tools, and support for large-scale structural models.

2.4. Modal vibration configuration

Modal and random vibration analysis determines the dynamic behavior of an external store structure under excitation from fluid-structure interaction and the structure's natural frequencies. The evaluation inputs are chosen following the provisions of the MIL-STD-810F standard. This regulation provides a practical introduction to existing theories on vibration analysis. The terminology, methodology, requirements, failure criteria, and descriptions are used according to this standard. The procedures in this standard are based on international regulations to determine the vibration exposure of a turboprop aircraft's life cycle.

Natural frequencies are the intrinsic response of any structure to external excitation when allowed to vibrate freely without damping. These frequencies depend on the location at which the structure of interest is excited. Knowing the natural frequencies of a body is crucial in structural design, as this information can be used to avoid drastic failures caused by resonance.

External Storage's natural frequencies and the spectra at which the Embraer EMB 314 aircraft vibrates in its ventral fuselage station must avoid resonance effects. That is why it is necessary to ensure that the external store natural frequencies are not coincident with the excitation frequencies of the aircraft that oscillate between 162 Hz and 180 Hz. It can be achieved by designing the external storage structure to have natural frequencies outside the range of the aircraft's excitation frequencies. The external storage structure was configured with two fixed supports (Figure 2), representing the boundary conditions applied in the assembly simulation without damping. The governing equation of motion for modal analysis is shown in Equation (1):

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=f(t), \quad (1)$$

where $[M]$ is the mass matrix of the system, $[C]$ is the damping matrix of the system, $[K]$ is the stiffness matrix of the system, and $\{u\}$ is the position vector with its respective derivatives that give velocity and acceleration (Zhang et al., 2022). As previously stated, the modal analysis is performed in a free and undamped system, meaning the damping matrix, $[C]$ is zero. Simplifying the governing equation of motion to the homogeneous equation shown in Equation (2):

$$[M]\{\ddot{u}\}+[K]\{u\}=\{0\}. \quad (2)$$

Equation (2) is of little interest in the time domain, representing a static or constant speed structure with zero acceleration. For this reason, it is analyzed in the frequency domain, where it is assumed that each point of the structure is experiencing harmonic motion. This transforms the variables from displacement and acceleration to amplitude, angular frequency, and phase angle. The position or displacement vector is then described by Equation (3):

$$\{u\}=\{\phi\}_i \sin(\omega_i t+\theta_i). \quad (3)$$

Differentiating Equation (3) twice in terms of time, the acceleration of each point is obtained in Equation (4):

$$\{\ddot{u}\}=-\omega_i^2\{\phi\}_i \sin(\omega_i t+\theta_i), \quad (4)$$

where $\{\phi\}_i$ is the amplitude, ω_i is the angular frequency, and θ is the phase angle. Substituting Equation (3) and Equation (4) in Equation (2), result in the eigenvalue problem described in Equation (5).

$$([K]-\omega_i^2[M])\{\phi\}_i=\{0\}. \quad (5)$$

The mode shape associated with each natural frequency is represented by the eigenvector $\{\phi\}_i$. The mode shape describes the deformation pattern of the structure at a particular natural frequency. The eigenvalue problem is solved using finite elements to compute natural frequencies and their modes.

Knowing the natural frequencies of a body is crucial in structural design, as this information can be used to avoid drastic failures caused by resonance (Khadse & Zaweri, 2015). In this case, it is desired to prevent resonance between the natural frequencies and associated mode shapes of the external storage structure and the frequencies at which the Embraer EMB 314 aircraft vibrates in its ventral fuselage station.

2.5. Random vibration setup

Once modal analysis is complete, the data and boundary conditions are used to perform a random vibration analysis using the PSD method. Random frequency response is a type of dynamic analysis that can only be described statistically (Bonte et al., 2007). The intensity of the random oscillation is described in the frequency domain by the PSD function, a mathematical concept describing how a signal's vibration is distributed in frequency (Youngworth et al., 2005). The PSD method is used because the load does not behave harmonically. The load data is dispersed, and the PSD method allows us to relate the mean squares of the signal RMS to its frequencies. The PSD function is described by the constraints shown in Table 3.

Table 3. Vibration exposure for turboprop aircraft

Mounting location	Vibration level $L_0\left(\frac{g^2}{Hz}\right)$
Empennage and pylon	0.6
Relevant points	
f_0 = 179.55 Hz	
$f_1 = 2 \times f_0$	
$f_2 = 3 \times f_0$	
$f_3 = 4 \times f_0$	

On propeller airplanes, vibration conditions are dominated by harmonics and relatively strong amplitude spikes at the propeller frequency. Variations in engine speed produce a bandwidth of frequency variation in the spikes. Wide-band vibration occurs at lower levels across the spectra. This wide-band vibration is primarily caused by the boundary layer flowing over the aircraft. Propeller-induced vibration is the primary noise source for components mounted in propeller aircraft. The vibration frequency spectra exhibit narrowband spikes superimposed on a broadband background. Propeller-induced vibration is the primary vibration source for components placed in propeller-powered aircraft. The spectrum of frequencies consists of narrowband spikes stacked on top of a broadband background.

Because turboprops have rotating components (engines, gearboxes, shafts), the background spectrum combines multiple lower-level periodic components and random sources. The spikes are caused by pressure fields passing in tandem with the propeller blades. These occur in tiny bands centered on harmonics and the propeller passage frequency (the number of blades times the propeller rpm). Most modern propeller aircraft, such as the Embraer 314 of this study, are constant-speed models; this means that the rpm is kept constant, and the power is changed by altering the fuel flow and using blades, vanes, and propellers with variable pitch.

The aircraft maintenance manual (AMM) of the aircraft (Embraer, 2012b) specifies that the standard mode of operation of the plane is at a propeller rotational speed (Np) of 1995 rpm. Additionally, the Embraer EMB 314 aircraft operates at a frequency of 35.91 Hz at 2154.6 rpm with a five-bladed propeller. The propeller pitch frequency (f_0) is calculated by multiplying the number of blades by the propeller rpm.

To compute the PSD function, it is necessary to calculate several mathematical equations; the procedure will be explained and summarized here according to the procedure established by Preumont (2013). Let's consider a signal (x) in the time domain (t), first it is necessary to determine the frequency spectra by using the Fourier series, which represent the signal as a sum of cosine and sine waves, as shown in Equation (6):

$$x(t) = \bar{x} + \sum_{i=1}^{\infty}\left[C_i \cos(2\pi f_i t) + S_i \sin(2\pi f_i t)\right]. \quad (6)$$

The term $\bar{x}$ represents the mean value of the amplitude, f_i refers to the frequency associated and the coefficients C_i and S_i are computed according the Equation (7) and Equation (8) respectively:

$$C_i = \frac{2}{N}\sum_{n=0}^{N-1}[C_i \cos(2\pi f_i t_n)\cdot x_n]; \quad (7)$$

$$S_i = \frac{2}{N}\sum_{n=0}^{N-1}[S_i \sin(2\pi f_i t_n)\cdot x_n]. \quad (8)$$

The values of the PSD are the mean square amplitudes of the FFT at each frequency divided by the frequency spacing, $\Delta f = 1/t$. The PSD function is then calculated with Equation (9).

$$PSD_x(f_i) = \frac{0.5\left(C_i^2 + S_i^2\right)}{\Delta f};\ i = 1,\ldots,\frac{N}{2}-1. \quad (9)$$

The Fourier coefficients C_i and S_i are calculated using the FFT. The mean square amplitude is given by the formula $0.5\left(C_i^2 + S_i^2\right)$. By dividing the mean square amplitude by Δf, the PSD is normalized to a 1 Hz bandwidth. Given that the frequency resolution $\Delta f = 1/T = 1/(N\Delta t)$. Then, mean, standard deviation and mean square values are considered to process with statistical results and approaches under a Normal Gaussian Distribution (Guzik & Więckowska, 2023).

3. Results

3.1. Aerodynamic results

The fluid-structure interaction (FSI) generates pressure differences over the surface of the external store due to aerodynamic effects. This influences the structural behavior of the POD at every phase of flight. In the event of an emergency or an evasive maneuver, the external store is designed to be securely detached from the aircraft. In this case, a delay must be configured in the lug suspensions, and the pod must generate opposing forces.

Table 4. Critical aerodynamic loads

Phase of flight	Pressure [Pa]	Force [N]	Z-moment [N*m]
Takeoff	59696.54	268,21	74.63
Landing	54433.98	126,33	18.05
Cruise at 1500 [m]	70502.50	588.00	153.24
Cruise at 3000 [m]	67648.26	552,65	126.11
Cruise at 4500 [m]	66635.31	332,84	104.26

Critical flight conditions are estimated in takeoff, landing, and straight and level flight conditions at 1500−, 3000−, and 4500-meters flight height, within the structural limits of the EMB 314 aircraft, to determine the most critical pressures and forces and moments that the structure must support. The CFD simulation within a subsonic turbulent regime under the effects of atmospheric pressure, density, and temperature leads to the results shown in Table 4.

Table 4 presents the maximum values of force, pressure, and moments that the external store structure will withstand in the different flight phases. The pressure difference variables allow us to analyze the phenomenon of mass and moment transfer following the Bernoulli principle, which establishes that changes in velocity are proportional to dynamic pressure and altitude.

The aerodynamic analysis revealed that the most critical aerodynamic pressure occurred during the cruise phase at 1500 m, with a value of 70,502.5 Pa and a corresponding moment of 153.24 Nm. This confirms that, at cruise altitude, the external store experiences the highest dynamic pressure due to airspeed and lower air density. These results are critical for identifying the structural zones subject to maximum aerodynamic stress. Importantly, this allows for targeted structural reinforcement, particularly in the nose region, where pressure concentration is most significant. The alignment of these peak values with expected aerodynamic loading scenarios confirms the CFD model's validity for use in subsequent structural simulations.

3.2. Static structural results and analysis

For the structural analysis, the loads applied to the structural core of the external store were considered, as these will be transferred from its coating to the pylon of the fuselage station. The effects generated by the forces, pressures, and moments applied to the structure were simulated using finite elements to determine where the most critical stresses and displacements occur.

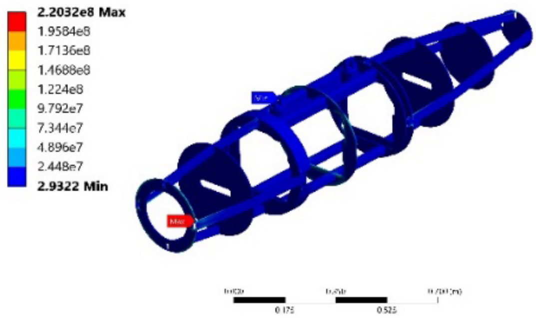

Figure 4. Equivalent stress distribution

The FEA simulation's equivalent stress result is displayed in Figure 4; the graphic illustrates that the highest equivalent stresses are concentrated in the nose of the aircraft's external store, where an equivalent maximum stress value of 220 MPa is estimated at the first rib of the head of the pod. This aligns with the expected loading configuration, where downward aerodynamic forces generate the largest bending moment at the front end of the pod.

The stresses are relatively low in the structure's core and near the suspension lugs. The material capacity and the fixed points located at this place gave the structure an excellent embedment to the fuselage station. The overall stress distribution is in blue, which means it is inside the elastic region of the material's capacity.

The stress distribution remains within the elastic range of both stainless steel and carbon fiber materials used, ensuring that the structure does not undergo plastic deformation under critical flight conditions. Moreover, as seen in Figure 5, the maximum total deformation of 1.26 mm confirms structural rigidity and suggests that the design maintains alignment tolerances for payloads. This is because the structure as a whole act like a beam suspended on two fixed points, and a load is applied downward. The physical phenomena occur as expected as the lift is negative, and the nose of the Pod tends to go downwards. These static outcomes validate the mechanical design by demonstrating that the structure can safely withstand the worst-case aerodynamic loading scenario using a conservative safety factor.

3.3. Modal vibration results and analysis

Modal analysis was performed with a maximum of 199 modes, which have their respective natural frequencies at which the structure vibrates. Table 5 shows the highest values of mode and frequency that contribute to the excitation of the structure and their associated data. Each mode and frequency have participation factors, measuring the mass moving in the X, Y, and Z directions. A higher value in a participation factor indicates a higher probability of excitation of forces in that direction.

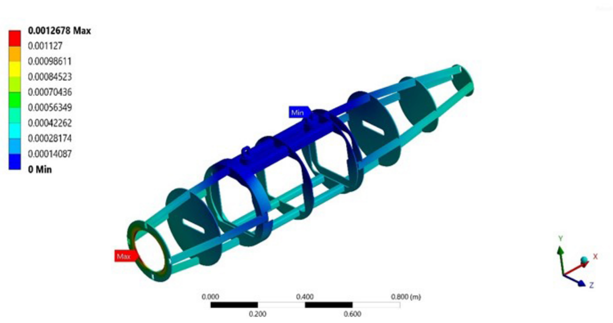

Figure 5. Total deformation

Analyzing the ratio of effective mass to total mass, it can be estimated that the modes that contribute the most to the structural response in a particular direction are the first three modes. The effective mass estimates the mass associated with each mode and provides insight into the design's response to different dynamic loading scenarios. In this study, more than 80% of the mass participation is being modified due to vibrational effects. This is important because the modal simulation's frequencies, modes, and shapes are the input to the random vibration analysis.

Mode 2 considerably affects the structure in the X and Y axes; it happens at a frequency of 71.20 Hz, participating with a significant amount of the structure's mass. On the Z axis, mode one approximately represents half of the structure's mass moving at 65.46 Hz. Modes 3, 8, and 9 also considerably affect the structure's natural vibration.

Comparing the modal analysis results in Table 5 to the range of aircraft operating frequencies, it can be determined that no resonance occurs between the aircraft's operating frequency range and the structure's natural frequencies.

Table 5. Natural frequency in X, Y, and Z planes

Axis	Mode	Frequency (Hz)	Participation Factor	Effective Mass	Eff Mass to Total Mass
X	2	71.20	1.117	1.249	0.041
	8	216.42	1.380	1.906	0.063
Y	2	71.20	2.001	4.004	0.133
	9	232.42	2.477	6.139	0.205
Z	1	65.46	3.824	14.630	0.488
	3	100.22	–1.031	1.063	0.035

Modal analysis identified critical natural frequencies at 65.46 Hz (Z-axis), 71.20 Hz (X and Y-axes), and above 200 Hz for higher modes. These are significantly separated from the aircraft's operating frequency range (162–180 Hz), confirming that the external store's modal response avoids harmful resonance under flight conditions. The effective mass participation for the first three modes accounts for over 80% of the dynamic response, meaning that the model effectively captures the structure's principal vibration modes. This is critical for ensuring vibration resilience, as undetected modes can lead to unforeseen failures. Thus, the modal configuration is not only compliant with MIL-STD vibration criteria but also strategically designed to isolate structural frequencies from known excitation bands.

3.4. Random vibration results and analysis

Using the findings of the modal analysis and the PSD acceleration values for random vibrations shown in Table 3 as inputs, the following results have been obtained for displacements and stresses applied to the structure due to random vibration loading.

To estimate the maximum displacements of the structure in the X, Y, and Z directions, a probability of 99.73% was considered to determine that the structure deformations are less than the values presented in Table 6.

Table 6. Maximum displacements due to random vibration

Axis	Maximum deformation [mm]
X-axis	1.22
Y-axis	0.89
Z axis	0.34

Similarly, equivalent stress is calculated due to the random vibration applied to the external store structure. With a 99.73% probability, the equivalent stress value is less than the values mapped in the structure. The maximum equivalent stress found is 197.32 MPa, as illustrated in Figure 6.

Comparing the maximum stress value found in the structural simulation due to random vibration analysis with the tensile yield strength of the prepreg carbon fiber, a safety factor of more than two is estimated.

Another relevant parameter from the random vibration analysis is the PSD response. Figure 7 displays the PSD response of the structure for displacements against frequency. The peaks in this figure represent the points where the largest displacements occur at the specified frequency. As it is seen, the highest resonance peak concerning random vibration excitation occurs between 60 Hz and 110 Hz. Additionally, the range between 150 Hz and 200 Hz presents values of harmonic response. A good indicator to tolerate aeroelastic damage during the external store operation. Therefore, it is demonstrated that in the cases of reso-

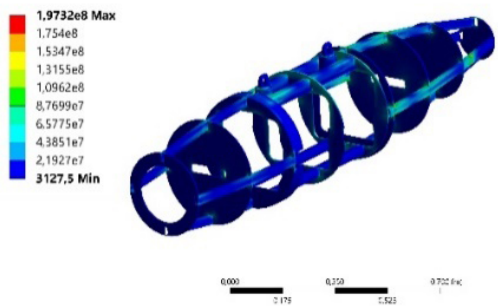

Figure 6. Equivalent stresses of random vibration analysis

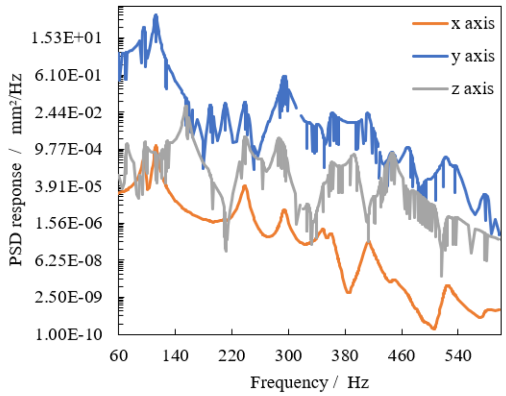

Figure 7. PSD Response displacement due to random vibration

nance between the POD structure and the aircraft with its random vibration, the displacements generated in the structure will be much less than what the structure resists in any of its materials.

Maximum total stresses and deformation comparison is shown in Table 7. Random vibration analysis results indicate that the maximum stress applied to the structure is 197 MPa. In contrast, the finite element analysis simulation gives an approximate value of 220 MPa. It is interpreted that those results are inside the elastic region of the materials used, which means that during both procedures, the intrinsic requirement of the structure design is to withstand the loads the external storage will suffer during a critical operation. Both computations must consider a safety factor of at least 1.5 to get a certification for modification. The significance of stress convergence lies in the field of aerospace structural design because it allows the corroboration of the static and dynamic response of an aircraft alteration by two different approaches whose results lead to conservative and approximate results.

Table 7. Maximum total deformation and stresses comparison

Mechanical Properties	Finite Element Analysis	Vibration Analysis
Maximum stresses	220 MPa	197.32 MPa
Maximum total deformation	1.26 mm	1.21 mm

Correspondingly, the maximum total deformation of 1.21 mm is estimated with the PSD method, and 1.26 mm is obtained with FEA computations. The structure's deformation is essential because it predicts the likelihood of failure under specific loads. One of the main goals of the present research is to identify the level of convergence between FEA and vibration outcomes; as a conservative fail-safe design, the most critical situation is analyzed in detail and determined pretty similar results for the maximum total deformation obtained with both different methodologies, all of them inside the flexural capabilities of materials and components. As in the previous case, the convergence of maximum deformation is significant for designing aerospace structures, leading to findings that allow the design and production of robust and reliable components, saving sources, and obtaining aeronautical products that meet the design requirements for certification.

Table 8. Vibration analysis results and test flight comparison

Mechanical Properties	Vibration Analysis
Maximum total deformation	1.21 mm
Natural frequency peaks	60–110 Hz
Aircraft's natural frequency	162–180 Hz

Table 8 displays the vibration analysis for comparison. The results of this study show that the external store structure is most susceptible to vibration at frequencies between 60 Hz and 110 Hz. The external store is designed at different frequencies from the aircraft's operating range to avoid resonance effects, which oscillate between 162 Hz and 180 Hz. This result confirms that the structure will operate at different excitation levels, significantly contributing to the dynamic behavior of any component installed in the aircraft or alterations required to change the mission profile outside the resonance spectrum.

3.5. Discussion

The results of this study highlight the importance of integrating modal and random vibration analysis into structural design for aircraft external stores. The close agreement between maximum stresses and displacements obtained from FEA and PSD-based vibration analysis – with a deviation of less than 10% – validates the accuracy and reliability of the proposed methodology.

The critical observation is that no resonance occurs between the external store's natural frequencies and the aircraft's operational frequency range (162–180 Hz). This confirms the success of the design in avoiding destructive harmonic excitation, one of the most important structural risks in dynamic flight environments. Additionally, the structural integrity under combined aerodynamic and vibrational loading indicates that the materials and configuration are well within safe operational limits. The results support the use of hybrid structures combining stainless steel and carbon fiber for optimal weight-to-strength performance.

These findings are particularly relevant for military and multipurpose aircraft, where mission adaptability demands reliable and certifiable pod designs. The convergence of static and dynamic simulation results also reduces dependency on extensive physical testing, streamlining development timelines and reducing cost. However, it is noted that the study assumes ideal boundary conditions and does not yet account for temperature variations, fatigue accumulation, or aeroacoustic factors, which may be addressed in future research.

4. Conclusions

A validation was done to determine the vibration response of the aircraft's external store during a critical operational phase; the aircraft mission was considered the most critical situation determined during the studies of the present study, defined by its constraints and boundary conditions given due to the performance and atmospheric conditions. During this situation, the total maximum displacements found were determined. As a result, a displacement of 0.9 mm was reported as the most considerable amplitude detected. This data got a good approximation with the results found with the FEA and PSD methods; it is less than the value considered in the numerical simulation because they consider load factors that increase the structure's response. However, the displacement detected experimentally presents several confirmations and validations of the proposed method and its relationship with the different design approaches.

Frequency peaks represent the most critical resonance frequencies the structure will support during a dynamic excitation. The vibration analysis shows peaks fluctuating between 60 Hz and 110 Hz and harmonic patterns presented after this excitation. On the other hand, the flight test result shows critical excitation peaks between 75 Hz and 110 Hz. This good convergence helps us understand the structure's aeroelastic behavior to detect signal behavior, resonance peaks, and harmonics. These are valuable insights to prevent fatigue damage and avoid structural failures.

Another implication of this design method is the identification of modes, frequencies, and structures most susceptible to failure due to vibration. This helps to identify areas where the stresses and displacements caused by vibration can be reduced by modifying the materials or geometries, an important alternative for detecting damage at early stages. The suggested design for the external storage can sustain the expected flight load requirements, according to the findings of the vibration study, and has been modified as desired in the design phase.

Compared to conventional numerical approaches, analyzing power spectral density (PSD) data allowed an accurate representation of the flying loads and vibration excitation. As a result, structural response was predicted with high accuracy. The PSD data-based analysis approach is verified against experimental findings from a designed external store. Good agreement is seen in the results, indicating the precision and reliability of the selected method. The convergences with the finite element analysis validate the results and confirm a correct design. The materials chosen for the external store must have a high strength-to-weight ratio to provide aeroelastic resistance under the applied loads.

Several previous studies demonstrate how fluid-structure interaction influences structural design by utilizing a variety of methodologies, such as Lattice methods, Hamilton's variational principle, and Euler equations. This study places its attention on modal and random vibration analysis utilizing the PSD approximation to estimate a structural design technique for an aircraft's external storage; the results present a convergent approach for designing this type of aeronautical component, relating a methodology to get stresses and displacements inside the material's capability and avoiding resonance effects in the structure.

The suggested design process for external stores may be extended using PSD data to other aircraft structures with similar load conditions. It can be possible to develop the methodology for other aircraft components, such as fuel external tanks and certain kinds of weapons installed in the fuselage pylon station. The proposed process can also optimize aerospace structures, find lighter and more efficient structures with better dynamic performances and capabilities, and design them to meet certification requirements inside a safe and robust design philosophy.

One limitation of this study is that it was conducted for a turbo propeller aircraft with an external store located at the fuselage station. The aeroelasticity effects of the wings could alter the behavior of the structure. Future research could investigate the proposed methodology at different wing stations and the effects of aerodynamic forces on the external store performance. In the same way, future works could analyze the presented relationship for other flight regimes, such as supersonic and transonic flight phases. In addition, the methodology focused primarily on mechanical properties behavior; future studies should consider other design objectives such as cost, manufacturability, and environmental impact.

Accurate and dependable findings were obtained from the PSD method, indicating that it is suitable for practical use. This approach is easily adaptable to different phases of the design and development process, offering understanding inputs for improved design. As the PSD function and vibration analysis is a study to understand the behavior of the structure under dynamic loads, its results could be used for advanced fatigue analysis, where the inputs of vibration response are considered and estimated in advance according to the location, function, and operational conditions desired. Further analysis could assess fatigue life considering complex PSD models and other mechanical properties such as ultimate strength, impact resistance, or aeroacoustics optimization.

It is noted that the PSD function provides several good approximations and estimations of the desired results. However, PSD functions and data are generalized for a specific aircraft type, location, and structure. Further research is needed to complete this gap by developing more accurate and specified PSD models for different flight conditions, aircraft, and mission profiles for different component alterations or modifications. The possible outcomes will serve as a good reference pattern, simplifying the aeronautical component design.

In summary, the results of this research show that the PSD approach is a practical approximation tool for evaluating the vibrational effects on aircraft external stores during both design and operational phases. The PSD method's results converge from experimental simulations and finite element analysis. This redundant methodological approach enables conservative approximations in dynamic structural design, reaching a margin of error of 4% difference in the maximum total deformation and 9% in resulting total stresses. Overall, the research offers a conservative aircraft external store design method based on modal and random vibration analysis. This method helps to understand the structure's dynamical behavior, avoid resonance effects, resist aeroelastic forces, minimize vibration, prevent structural damage, and save computational and economic resources, increasing safety during the design phase.

Acknowledgements

The authors would like to thank the Center of Research and Development of the Ecuadorian Air Force for their assistance and support to the present study.

Author contributions

Criollo Linker conceptualized the study, collected information for analysis, and wrote the initial draft of the manuscript. Mena Carlos conducted the simulations, literature review and created the data visualizations. Juan Brazalez and Fabricio Medina interpreted and analyzed the information, and Marco Zurita validated the results, and edited the paper.

Disclosure statement

The authors declare that no competing financial, professional, or personal interests or benefit they have arising from the direct applications of this research.

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