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*Review Article*

# TOPOLOGY OPTIMIZATION METHODS FOR MORPHING AIRCRAFT DESIGN: A REVIEW

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**Abstract.** Current aeronautical research efforts are increasingly focused on weight reduction and the integration of advanced materials analysing dynamic properties. These efforts encompass cellular structures, flexible skins, and modifiable primary and secondary structural elements (e.g., wings). The development of technologies for morphing aircraft design enhances aerodynamic performance and structural efficiency, thereby optimizing the mechanical design of these systems. The authors provide a comprehensive review of the current state of topology optimization methods in morphing aircraft design, highlighting the number of publications in this field and identifying the key journals contributing to this research. It also offers an in-depth analysis of the Solid Isotropic Material with Penalization (SIMP) method, the Evolutionary Structural Optimization (ESO), Bidirectional Evolutionary Structural Optimization (BESO), the recent Proportional Topology Optimization (PTO) and evaluates their effectiveness in achieving efficient designs. Additionally, the review discusses of future challenges and potential advancements in topology optimization for morphing aircraft, offering a thorough overview of the field.

**Keywords:** topology optimization, morphing aircraft, solid isotropic material with penalization, SIMP, evolutionary structural optimization, ESO, bidirectional evolutionary structural optimization, BESO, proportional topology optimization, PTO.

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## 1. Introduction

Aeronautical engineering is constantly evolving, driven by a growing interest in innovative design methodologies that are expected to revolutionize the performance and efficiency of aircraft structures. In modern engineering, lightweight design is both a requirement and an objective in the field of structural design. Achieving this necessitates the use of structural optimization approaches, including shape optimization, size optimization, and Topology Optimization (TO), as mentioned by Cheng et al. (2021). Among these, TO has emerged as a particularly promising approach in the design of morphing aircraft, as it systematically explores the distribution of materials within a design space, allowing for optimal structural configurations while adhering to specific constraints. By maximizing system performance and considering specific loads, boundary conditions, and constraints. TO can significantly enhance the efficiency and performance of aircraft structures. This technique holds immense potential for the future of aircraft design, with the capability to push the boundaries of what is achievable in terms of performance and efficiency.

Since the early 20th century, aircraft designers have continuously sought methods to enhance lift during critical phases such as landing and takeoff, which has presented ongoing optimization challenges. The Wright brothers were pioneers in this area, utilizing variable wing curvature for lateral control, as noted by Gupta et al. (2022). However, to avoid different aeroelastic phenomena such as divergence or flutter, due to the increase in the weight of the aircraft and the cruising speed, which produces an increase in the structural rigidity of the wing, initial solutions focused on isolated control surfaces, such as flaps and ailerons, rather than wing torque. By the late 1970s, researchers began exploring the concept of wings with variable shapes based on two key approaches: the implementation of flexible wings and the active control of curvature along the span to exploit aeroelastic forces for desired deformations, as mentioned by Das et al. (2022). Studies have also examined varying wing geometries inspired by bird flight and mission-dependent modifications, according Sofla et al. (2010), Yousaf et al. (2021). Zhong et al. (2022) discussed objectives for controlling shock waves, turbulence, vortex formation, and laminar flow separation during transonic flight using surface modifiers or advanced control systems. The leading technology in this area focuses on altering the aerodynamic profile's curvature through the deployment of flaps or slats to manage aerodynamic forces and moments (Ermakova & Dayyani, 2017).

According to Cavalieri et al. (2020); Zhang et al. (2021), the concept of morphing is highly considered today by contemporary aircraft designers. A main component of morphing aircraft is the wings, which enhance overall flight efficiency by adapting to various flight conditions. These wings can modify their shape to optimize performance across specific speed ranges, reduce vibration, increase aeroelastic efficiency, and minimize drag. Additionally, they contribute to active flutter suppression and improved aircraft maneuverability, as discussed extensively by Ajaj et al. (2021), Barbarino et al. (2011), Olivett et al. (2021), Ouyang et al. (2021), Selim et al. (2020), Wang et al. (2018). Morphing wings can also enhance stealth capabilities by reducing the presence of irregular rudder surfaces. The concept of morphing originates in nature, where the adaptive characteristics of plants, leaves, insects, birds, and fish have inspired scientists and engineers to develop structures that adjust based on operating conditions, as mentioned by Jha and Dayyani (2021), Lumpe and Shea (2021). Esfarjani et al. (2022) explain that "morphing" refers to technologies that enhance a vehicle's performance by adjusting specific characteristics to better align the vehicle's state with the environment and the task at hand.

Continuing with morphing concepts, also is defined as "one that is able to adapt its external shape" as noted by Özgen et al. (2010), holds significant promise. Specifically, wing morphing enables the exploration of an aircraft's aerodynamic potential by adapting its shape to various flight conditions encountered during a mission profile. Aeroelastic deformations have an essential role in achieving improved performance and maneuverability. By allowing the wing to respond to aerodynamic forces, it is possible to improve structural efficiency. Figure 1 illustrates the flight conditions, with each axis representing a metric performance related to specific aeronautical scenarios. The current challenges in transforming air vehicle design include managing complexity and weight, particularly concerning

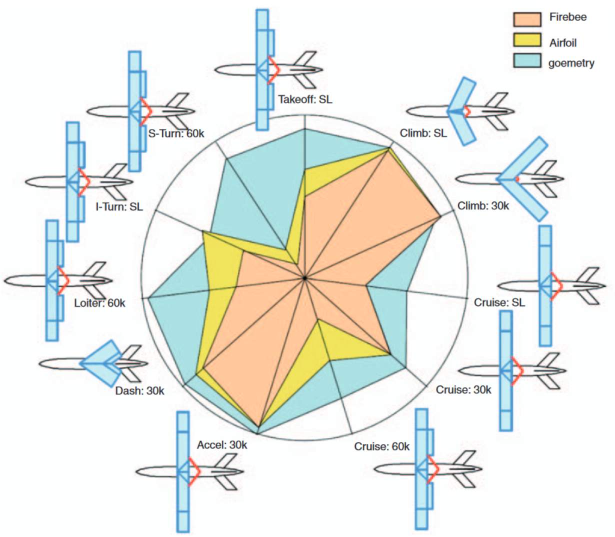


**Figure 1.** Flight conditions (Barbarino et al., 2011)

energy consumption. This is especially important when considering distributed actuation concepts for developing structural mechanisms integrated with flexible skins.

The concept of morphing assume the existence of an adequate flexible skin as noted by Bai et al. (2017). The average skin strain level ranges from 2% to 5% when varying the airfoil chord length or camber; however, for wing morphing, the flexible skin strain level needs exceed 10%. This requirement presents a challenge due to conflicting demands: the skin must be soft enough to allow shape variation while also being rigid enough to resist aerodynamic loads and maintain the desired shape. Additionally, flexible skin can be one-dimensional or multidimensional, depending on its use and application, as described by Barbarino et al. (2011). The concept of flexible skin, as explored by Kollmann et al. (2020), Valdevit and Bauer (2020), necessitates the consideration of metamaterials. These materials are typically optimized by selecting the appropriate volume fraction and architecture, which is crucial in designing materials with unprecedented properties. These properties include a high stiffness-to-weight ratio and an (NPR) negative Poisson's ratio, are generally achieved through a combination of intuition, experimentation, bioinspiration, and topology optimization, as discussed by Dalaq and Barthelat (2020), Gao et al. (2018), Kumar et al. (2021), Vangelatos et al. (2019), Wegst et al. (2015). Topology optimization allows for the design of materials that accommodate specific loads, constraints, and boundary conditions, as originally explored by Bendsøe and Kikuchi (1988). This approach involves identifying the optimal material distribution that maximizes system performance while adhering to design constraints, as further elaborated by Bendsoe and Sigmund (2013).

For an extended period, the pursuit of higher flight altitude and speed has necessitated improvements in structural stiffness, as considered by Narváez-Muñoz et al. (2023) due to challenges such as aeroelastic issues and wing deformation. Aircraft operate under various conditions, including cruise, takeoff, landing, and maneuvers, which can result in reduced fuel efficiency and increased operational costs, as noted by Cramer et al. (2019). Consequently, designers and researchers are now re-evaluating aircraft structural strategies, focusing on the development of new materials, design methods, and processes that enable more effective and precise wing morphing configurations, as discussed by Haro et al. (2023). Additionally, some academics such as Zhang et al. (2021) have explored the creation of adaptive components, including blades and wings, inspired by piezoelectric actuators, leading to the development of active deformable devices.

Considering structural improvements, over the years, advances in flexible skins, particularly in cell frameworks, have demonstrated that the overall performance of these framework can be controlled by adjusting the topologies and parameters of individual cells, ranging from nanometer to centimeter scale as discussed by Zheng et al. (2016). Similarly, Tapia et al. (2023), Zheng et al. (2014) have examined how lattice networks of cellular structures, when fabricated using polymers, ceramics, or metals, can achieve ultra-rigid mechanical properties. Thus, the Topology Optimization method can serve as a foundational step in the design of flexure elements. Subsequently, this method can refine material distribution using cellular material topology, enabling a precise structure-property relationship and facilitating the generalization of the design, as explored by Arredondo-Soto et al. (2021).

Within this scope of morphing aircraft design, Thill et al. (2008) conducted a complete analysis of flexible skins, focusing on various novel material system concepts and technologies. Among the explored concepts and definitions is the use of a composite corrugated structure that alters skin panels in the trailing edge region to modify the chord and camber of a surface, as noted by Thill et al. (2010). Additionally, Peel et al. (2009), Wereley and Gandhi (2010) introduced a skin incorporating elastomeric matrix composites to detect significant area deviations. Olympio and Gandhi (2012) further investigated sandwich structures with shifting cores covered by a flexible face sheet, applicable to both high and low voltage settings, depending on the cell arrangement and core material. Furthermore, the use of segmented support structures has been explored to create materials with adjustable stiffness, as discussed by Alphonse et al. (2021), Mcknight et al. (2010), Olympio and Gandhi (2010). A prominent example of morphing aircraft technology is the Aérospatiale/British Aircraft Corporation's Concorde, which enhanced visibility during takeoff and landing through a hydraulically tilted nose, without compromising the aircraft's aerodynamic profile during supersonic flight, as explained by Chapkin et al. (2020).

Achieving morphing aircraft technology necessitates the use of Topology optimization, a field in constantly developing, incorporating advancements in computing, physics, mathematics, and mechanics. As noted by Lim et al. (2020), this approach focuses on reducing weight while increasing rigidity. According Negahban et al. (2024), TO is a computational design technique that iteratively generates mechanical designs to maximize performance based on specific conditions. Topology optimization is an excellent tool to solving structural engineering problems, optimizing for objectives such as maximum stiffness, minimum compliance, minimum weight, and reduced thermal potential energy. It is particularly effective in identifying the primary load transfer paths within a defined design space under given constraints, making it an invaluable approach in the field of structural engineering (Rao et al., 2024). Structural optimization is categorized into three primary areas: size optimization, which determines the optimal dimensions of structural components; shape optimization, where the structure's shape is first parameterized and then optimized; and topology optimization, which seeks the ideal three-dimensional material distribution to achieve a specific objective, according (Esfarjani et al., 2022).

Topology optimization has a crucial role in the development of morphing aircraft, where adaptability and multifunctionality are paramount. These aircraft can modify their shape, wing configuration, or surface properties in response to changing flight conditions, as noted by Barbarino et al. (2011). These adaptations offer numerous advantages, including improved aerodynamic efficiency, reduced fuel consumption, and enhanced maneuverability. This includes applications in Unmanned Aerial Vehicles, as discussed by Criollo et al. (2024). When addressing two- or three-dimensional problems, the optimal material distribution can only be determined by refining the design space for a continuous composite material description.

The history of structural optimization dates back to Michell (1904), where he introduced the concept of optimal truss design through a continuous description. This is considered the first work on optimization at multiple scales according Wu et al. (2021). Since then, researchers have significantly advanced the field over the past eight decades, with contributions from scholars like Lin et al. (2019). The first major theory that emerged was "Optimal Design Theory" as detailed by Abdi et al. (2018), introduced precise and systematic optimization of grid-type structures, further refined by Splichal et al. (2015). These developments have been particularly vital in designing aerospace assemblies that must endure dynamic loading, as evidenced by recent research.

Cheng and Olhoff (1981) later found that by optimizing the toughness of a plate by varying its width, stiffeners appeared when the design space was refined. Keng-Tuno (1981) concluded that infinitely many rigid reinforcing members could be produced in an infinitely fine mesh, necessitating restrictions on shape variation to prove the presence of a solution, as mentioned by Niordson (1983). The first significant developments in this field were made by Bendsøe and Kikuchi (1988), who used a model with square cells containing infinitely small rectangular holes, where mechanical properties were evaluated using numerical homogenization methods.

The optimal structures obtained through topology optimization can feature geometric patterns that vary spatially and across different length scales. However, due to the complexity of manufacturing these multiscale structures, the focus shifted in the late 1990s. Zhou and Rozvany (1991) introduced homogenization techniques at the nanoscale, leading to the development of a more isotropic material using the SIMP (Solid Isotropic Material with Penalization) method, as discussed by Mlejnek (1992). Subsequently, various methods were developed, such as the density-based approaches proposed by Sigmund (2001), the development of level-set methods by Allaire et al. (2004) and the innovative evolutionary procedures mentioned by Xie and Steven (1993).

Following this, Papanicolau et al. (1978), discussed the theory of homogenization in structural optimization, which bridges the gap between microscopic periodic composite materials and their homogenized properties at the macroscale. Simultaneously, N-rank laminates were developed to achieve the theoretical upper limits for maximum strain energy density, as explored by Francfort and Murat (1986). Through inverse homogenization, optimization problems can be designed to maximize the stiffness of the microstructure concerning functional stresses or deformations, accounting for variations in stiffness between the optimal energy limits and the designed microstructures, as noted by Träff et al. (2019). Furthermore, materials with unique mechanical properties, such as those with a negative Poisson's ratio, zero Poisson's ratio in honeycomb structures, maximum bulk and shear moduli, or enhanced resistance to buckling, can be developed, as discussed by Clausen et al. (2015), Gong et al. (2022), Qu et al. (2022), Sigmund (2000), Thomsen et al. (2018).

Alacoque et al. (2021) developed the homogenization-based topology optimization approach. As an alternative, they introduced, the SIMP (Solid Isotropic Material with Penalization) or power law approach, the material distribution is represented by a scalar pitch, relative density per component ($p = 0$ empty, $p = 1$ solid). Each element's material is assumed to be homogeneous and isotropic, with the material properties correlated to the relative densities through interpolation. This relationship is physically demonstrated, as in the case of a Poisson ratio $\nu = 1/3$ with a power $p \geq 3$, as discussed by Ferrari and Sigmund (2020) discussed.

Additionally, several methods can be optimized using density-based approaches, including:

- Level set-based methods.
- Evolutionary procedures.
- Projection and geometric transformation (Guo et al., 2014).

Topology optimization has advanced significantly in both industrial and academic applications, particularly as a means to reduce the weight of structural components, as noted by Saeed et al. (2020). Jensen et al. (2021) highlighted those studies on mechanisms with morphing wings have predominantly focused on two-dimensional (2D) models. Initially, explored the Solid Isotropic Material with Penalization (SIMP) method, which includes sensitivity filtering and optimality criteria. Subsequently, newer optimization techniques such as Evolutionary Structural Optimization (ESO) and Bidirectional Evolutionary Structural Optimization (BESO) were developed by Huang et al. (2018). Modern optimization features include data arrangement, evolutionary procedures, phase field methods, density-based methods, bubble methods, topological derivatives, and level set methods, as discussed in Sokolowski and Zochowski (1999), Townsend and Kim (2019), Wang et al. (2019), among others.

Biyikli and To (2015) introduced the Proportional Topology Optimization (PTO), which assigns design variables to elements proportionally based in stress problems and in compliance problems. This method imposes global constraints on the entire system to manage proportional distribution effectively. The PTO method adopts a modified SIMP approach, employs the maximum function as a stress constraint, and incorporates density filtering, including boundary conditions to adjust material distribution. Ullah et al. (2022) noted that the heuristic nature of PTO leads to its straightforward implementation in computer programs compared to alternative topology optimization methods. PTO offers quicker convergence and versatility for different loading conditions and multi-stage optimization, although it may have limited flexibility in handling complex design constraints or multi-objective problems. Variants of PTO, such as Proportional Topology Optimization for Stiffness (PTOs) and Proportional Topology Optimization for Compliant Mechanisms (PTOc), are tailored for specific structural optimization challenges.

Cheng et al. (2021) emphasized that numerous advancements have been made in topology optimization over the past decades. The primary goal of topology optimization is to achieve high-performance material distributions with distinct black (1) and white (0) structures that satisfy constraints. These approaches are generally divided into sensitivity-based optimization methods, such as the Optimality Criteria (OC) method, Successive Linear Programming (SLP) method, Convex Linearization (CONLIN) method, and the Method of Moving Asymptotes (MMA), and non-sensitivity-based optimization methods, such as genetic algorithms, ant colony algorithms, particle swarm optimization algorithms, neural networks, and the PTO algorithm.

This article delves into the complexities of topology optimization methods applied to morphing aircraft, including flexible skin and cellular structures. The primary objective is to analyze the principles and ongoing advancements in topology optimization methods within the context of morphing aircraft design. The research explores algorithm, design objectives, advantages and challenges associated with integrating topology optimization into the design process. This integration is important for mitigating aeroelastic phenomena and enhancing overall aircraft performance. By summarizing the methods employed, this study aims to establish a foundation for addressing future research challenges and advancing efforts in this demanding field.

## 2. Methods

According to the analysis, topology optimization is a complex tool but requires careful consideration of trade-offs, constraints, and practicality. It is essential to collaborate with experts to validate the results. Over the years, researchers have used four main methods to develop topology optimization in the context of morphing aircraft structures.

### 2.1. Solid Isotropic Material with Penalization (SIMP)

Since the development of the SIMP approach by Rozvany and Zhou (1991), initially oriented for large-scale problems, it has become the most used method in aerospace constructions. The topological optimization of continuous structures has been a focus for the past two decades, particularly in the aeronautical industry. Several researchers, such as Capasso et al. (2020), have implemented simple codes like 99 lines and top 88 to solve topology optimization problems. For instance, Krog et al. (2004) demonstrated its use in optimizing the internal edge ribs, exterior fixed attachments, and fuselage door intercostals of the Airbus A380, resulting in a weight savings of 1,000 kg per aircraft. Although Airbus UK reported optimizing the A380 wing rib topology, the design has not yet been commissioned, as explained by Maute and Reich (2004). Additionally, Gaspari and Ricci (2010) applied the SIMP method to two-dimensional problems, such as designing actuator mechanisms, transformable airfoils, and structural supports for the aircraft skin. These studies underscore the significance of aerostructural integration, ensuring that optimized designs perform effectively under aerodynamic loads and structural constraints. Guest and Moen (2010) explored the use of the SIMP method in three-dimensional design problems, such as optimizing a truss-reinforced spar within a wing, shown in the Figure 2. James and Martins (2008) further demonstrated the effectiveness of the isoparametric level set method in optimizing a three-dimensional wing box frame under fixed downforce constraints, highlighting the method's ability to manage maximum local stress with minimal reduction in design space.

James (2013) used a SIMP method to optimize the wing box topology and the aerodynamic shape simultaneously. Then, a level set method was used to optimize the wing of an interior structure, where aerostructural coupling was considered. The wing was optimized considering minimum agreement and maximum lift or identical to the aircraft's mass, that is, a multidisciplinary approach, as mentioned by Dunning et al. (2013).

Munk et al. (2016) introduced a topology optimization algorithm that uses this method, in a microstructure with a penalty (SIMP) for designing a wing structure optimized to reduce frequency excitation. The study achieved a 9% increase in the wing's fundamental frequency, raising it from 175 Hz to 190 Hz. Chang and Shen (2018) focused on optimizing the topology of cellular core structures in two-dimensional transformation skins. Their research applied the SIMP method alongside the Method of Moving Asymptotes (MMA) to achieve these designs. Qin et al. (2018), extended the approach beyond traditional topology optimization by incorporating functional elements like triangles and rectangles to evaluate the macroscopic negative Poisson's ratio (NPR) value. Their work targeted three objective functions: minimum compliance, minimum mass, and maximum compliance. The dynamic analysis revealed that metamaterial structures are particularly effective in reducing vibrations within the low-frequency

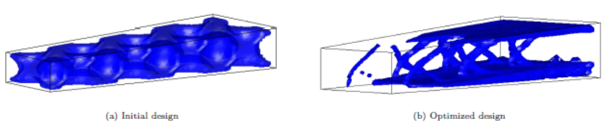


**Figure 2.** Initial and optimized designs for the three-dimensional wing optimization problem (James & Martins, 2008)

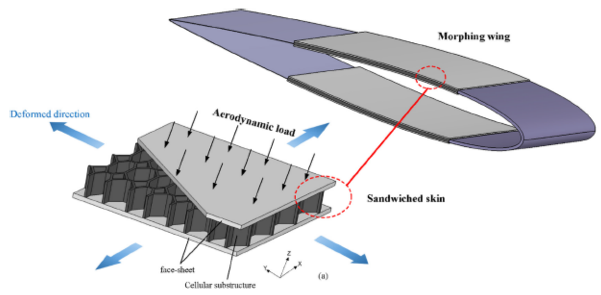


**Figure 3.** Schematic of a morphing wing, a three-layer shell model (Chang et al., 2020)

range, achieving a 36% reduction. Furthermore, Chang et al. (2020) explored the solid isotropic material interpolation scheme with penalty (SIMP), focused on density and is not inadequate to the number of cells, is used for the design of intercalated morphing skins and their mechanical properties are evaluated, this design procedure based on topology optimization in a cellular substructure for intercalated morphing skins is capable of optimizing structures, shown in Figure 3, considering three different flight conditions.

## 2.2. Evolutionary Structural Optimization (ESO)

Topological optimization, also known as evolutionary optimization algorithms, considering they do not have a firm theoretical basis, they imitate natural selection with evolutionary methods. Hence its name, Evolutionary Structural Optimization (ESO) was discussed by Tanskanen (2002). The ESO method was primarily announced by Xie and Steven (1997), based on the modest idea of gradual material elimination in the design to achieve a structure of optimal shape and topology as noted by Lampeas (2020). Huang et al. (2011) noted that this was developed to eliminate inefficient materials, resulting in a topology that evolves towards an optimum, presented by Huang et al. (2017). ESO is a structural optimization method that iteratively eliminates elements in a structural arrangement with minor importance or utilization. However, the ESO method eliminates elements without the option to recover those mistakenly deleted in some iterations, limiting its application according to Wang et al. (2011).

The ESO method has diverse applications, including magnetostatic and electrostatic fields, non-viscous and incompressible fluid flow, elastic torsion axes, and stable heat conduction. It has also been applied to aircraft morphing, as discussed by Steven et al. (2000). Das and Jones (2011) in their article developed the use of an ESO algorithm that has been modified for the design of an aerospace component in a bulkhead of a F/A-18 aircraft, consequential in a significant decrease in the weight of a structure and the adequate use of the material by obtaining a uniformly tensioned structure. Ikonen and Sobester (2016) focused this method on developing an instrument that allows for determining the space of topologies of alternative internal structures in a wing subject to stresses and buckling restrictions, allowing the structural mass to be minimized and evaluated through automated finite element analysis.

Furthermore, ESO method is used for the control design of a transformable aircraft with wings of variable-sweep founded on nonlinear systems switched and (ADP) adaptive dynamic programming, obtaining that the dynamic analysis for morphing aircraft is initially modelled as switched nonlinear systems in a lower triangular shape according Wang et al. (2019). Zhang et al. (2021) introduced ESO with an improved evolution rate to study the topology optimization design of CLD (constrained layer damping) behaviour for vibration clampdown of aircraft panels with a reasonably optimized topology geometry. Applying this topological optimization method biologically inspired aeroelastic, a morphing aerodynamic profile for a flight in supersonic conditions is generated using the ESO method instead of a gradient-based approach. Hodson et al. (2019) in this study mentioned, several morphing aerodynamic profiles have been planned so that, for a static angle of attack, they can acquire performance features similar to those of an airfoil at numerous angles of attack without camber.

## 2.3. Bidirectional Evolutionary Structural Optimization (BESO)

The development of early versions of the Bidirectional Evolutionary Structural Optimization (BESO) method enabled the recovery of elements that have been removed, as noted by Xia et al. (2018). This method simultaneously removes elements from regions of minimal efficiency and adds elements to the most efficient regions, optimizing the material distribution within the cell while adhering to volume constraints. Munk and Miller (2021) highlighted that BESO can address challenges involving both non-volume and multiple constraints. BESO offers greater flexibility in selecting initial design conditions and is characterized by reduced computational costs. The method has been applied to advanced structural designs, including stiffness optimization with multiple materials, displacement-related structural designs, structural natural frequency optimization, nonlinear periodic structures, and tunnel shape optimization. These applications are particularly relevant to morphing aircraft, as discussed by Zheng et al. (2016).

This optimization technique maximizes the volume or shear modulus in materials with auxetic materials or negative Poisson's ratio, which can deform the rotation of their ligaments, presented by Vogiatzis et al. (2017). The BESO optimization method was utilized to control the optimal thickness distribution of truss struts to improve the structural rigidity of an aircraft engine support and compare it with a non-optimized homogeneous truss structure; the method is easy to implement if you have Finite Element Assistant. Compared with the original solid material design, shown in Figure 4, the optimized lattice structure compacts the heaviness by about 75%, developed by Tang et al. (2015).

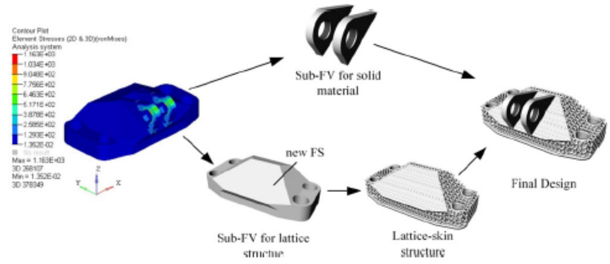


**Figure 4.** Aerospace component optimized (Tang et al., 2015)

Munk et al. (2015) demonstrated a novel bidirectional evolutionary optimization technique with aerothermoelastic connection for a hypersonic transport aircraft wing, considering a weight restriction of 20 tons without exceeding the limits of the material or causing buckling in the skin. Casas et al. (2016) employed the BEFSO method, a modification of the BESO method that incorporates fluid-structure interaction, to optimize a fixed wing on a single beam using an unstructured and irregular mesh. The primary objective was to maximize the energy absorbed by the structure while maintaining rigidity and adhering to material constraints. The bidirectional evolutionary structural optimization method relies on iterative geometric mean processing, with a focus on the vertical connector ear, a critical component of aircraft support structures. Munk et al. (2015) explored the application of the BESO method for designing aircraft components with load-bearing, finding that the topologically optimized component exhibited a 70% reduction in maximum stress, from 944 to 280 MPa, compared to the original design.

## 2.4. Proportional Topology Optimization (PTO)

Proportional Topology Optimization (PTO) is a variant of topology optimization used in structural design to determine the optimal material distribution within a given design space, under specific loads, boundary conditions, and constraints. The objective function is typically related to structural performance, such as maximizing stiffness or minimizing compliance, considering the inverse for stiffness, to determine where material should be placed or removed to achieve the best performance. Recently, Biyikli and To (2015) introduced a new non-gradient technique, in which the material is distributed into each finite element proportionally to the contribution of that element in the total value of structural compliance according to the objective function. Based on that, the algorithm was named Proportional Topology Optimization (PTO).

The Proportional Topology Optimization (PTO) method differs from gradient-based methods primarily in its optimizer, as the PTO algorithm does not require sensitivity information to update design variables. Biyikli and To (2015) presented a simple and efficient non-sensitivity method, known as PTO, to perform topology optimization for stress (PTOs) and compliance (PTOc) problems.

Their research demonstrated that PTOc achieves results comparable to those of the Top88 code without using sensitivities, while maintaining a similar level of efficiency. Wang et al. (2020) proposed four improved proportional topology optimization (IPTO) algorithms which are called IPTO_A, IPTO_B, IPTO_C and IPTO_D, respectively, to solve the minimum compliance optimization problem, avoiding the problems of numerical derivation and calculation of sensitivity in a beam. These algorithms were found to offer advantages over PTO and Top88 in terms of convergence and speed, resulting in optimized structural designs. Ullah et al. (2022) implemented a meshless method based on maximum entropy for two-dimensional linear elastic structures within the PTO framework, demonstrating that both PTOc and PTOs algorithms effectively handle significant topological changes and offer excellent optimization, which is particularly applicable in aeronautical design, as shown in Figure 5.

Various studies have further expanded the application of Proportional Topology Optimization (PTO). Nguyen et al. (2023) extended the PTO algorithm to address multi-material topology optimization for compliant mechanisms, where each material is proportionally distributed into elements based on its contribution to mutual strain energy. In compliance problems, "weak regions" are identified by elements with significantly negative mutual strain energy. Tran et al. (2023) introduced an enhanced non-sensitivity structural topology optimization method that, for the first time, incorporates virtual elements with unstructured polygonal meshes, using PTO for minimum compliance problems and compatible mechanisms. While, Wang et al. (2020) highlighted several advantages of PTO, such as not requiring sensitivity information, along with its efficiency, precision, and simplicity, they also noted limitations, including poor robustness, relatively low topological structure quality, and a limited ability to approximate the density variable to an optimal solution.

## 3. Discussion

Topology Optimization has evolved since the first concepts in 1988, especially in the technological part. However, its starting point is attributed to its first mention in 1904, "the representation of optimal truss design," to speak of topology optimization. It is directly related to a computational design method, where, in particular, it uses material such as small infinite squares to simulate homogeneous and isotropic material, the same whose objective is to reduce its weight, support the different aerodynamic loads, and the efforts to which it is subjected. Depending on the flight stages or the application to which it is directly improved or optimized.

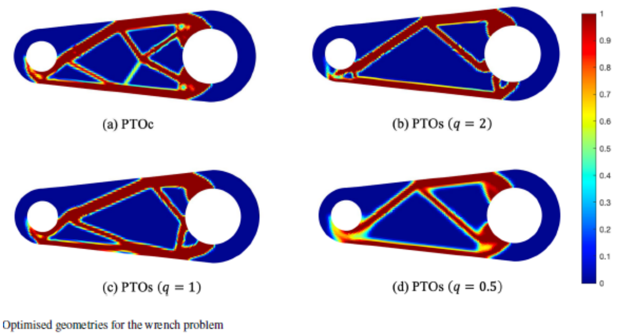


Figure 5. Optimized geometries using PTO (Ullah et al., 2022)

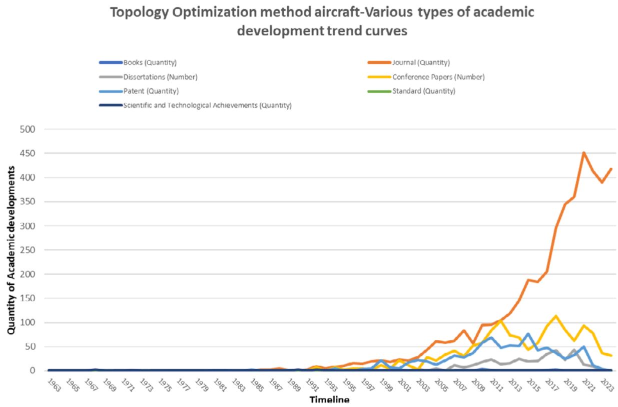


Figure 6. Academic development of topology optimization method (source: data from Super Star, 2024)

This particular research focuses on the studies carried out on topological optimization applied in morphing aircraft. The aim is to optimize the stiffness by modifying its dimensions and the reinforcements that appear in the design space at micro and macro scale. In this review, different methodologies have been found to apply optimization topology in morphing aircraft skin and development along with technology.

The Figure 6 offers a comprehensive outline of academic developments correlated to topology optimization in morphing aircraft over the past 50 years. The data obtained in the web page of Super Star (2024) encompasses publications, books, conferences, and patents. Notably, the expected contributions are in the form of publications, with a significant surge beginning in 2020. Specifically, there have been over 450 publications dedicated to topology optimization during this period.

The academic development of aircraft-related topology optimization methods related to aircraft structures over recent decades, noted that the production, mainly of academic journals on this topic, has experienced significant growth in recent years. During the 1960s, 70s, 80s, and 90s, academic production on topology optimization for aircraft structures was minimal. The annual publication count ranged from 0 to 10, indicating a nascent research stage. However, in the current years, there has been a paradigm shift. The field has experienced exponential growth, with an average of over 400 annual publications. This surge underscores the importance of topology optimization in advancing aircraft design and performance. Although conference articles are minor, they remain consistent over time, with contributions of specific aspects or novel techniques within the broader field. Researchers

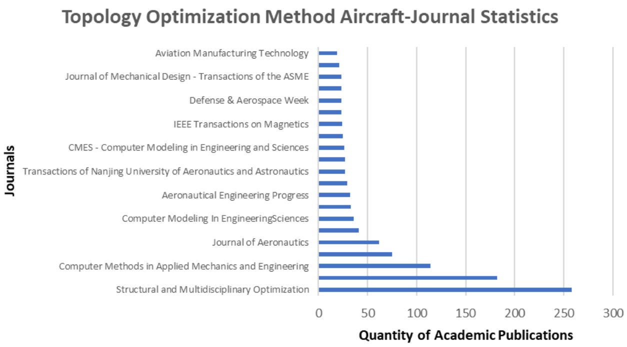


**Figure 7.** Journal statistics (since 1963 to 2024) of topology optimization methods

continue to explore innovative approaches; topics such as lightweight design, structural efficiency, aerodynamic performance, and material utilization govern the dissertation. Incorporating advanced computational gears and multidisciplinary optimization, interdisciplinary collaboration, data-driven methodologies, and a focus on real-world applications will shape the next research step.

Conducting this research, a comprehensive analysis of journal statistics reveals 1,123 publications explicitly addressing the topic of aircraft-related topology optimization methods. In particular, a significant part of the recent studies in this field originated from Chinese research institutions, playing an essential role in advancing research. The data was analyzed through the Superstar tool, which provides a centralized repository of academic publications, as seen in the previous Figure 7. The global research community continues to explore topology optimization methods, with China leading the way in more recent contributions, as shown in Figure 7.

This review required a classification of methods, which can cover how to solve problems or approaches when faced with a morphing aircraft. One of the first methods is the Solid Isotropic Material with Penalization (SIMP), presented in 1991. This method permits optimizing the material distribution, minimizing the rigidity, and subject to volume control. There are several successful examples of applying this method, among which is possible to name redesigning the internal and external edge ribs interior and exterior fixed attack for Airbus UK, design of transformable airfoils, truss-reinforced designs for a wing spar, internal structure of aircraft wings, wing box topology, design of optimized topologies of cellular structures focused on two-dimensional transformation skins, structures with negative Poisson's ratio or zero Poisson's ratio, among others.

The second one that has been detailed is Evolutionary Structural Optimization (ESO). This method was developed on the basis of natural selection or nature's evolutionary processes, hence its name to evolution, which always obtains an evolutionary topology. The principal idea of this technique is the elimination of inefficient materials or inefficient regions until the optimal material distribution is completed. ESO aids in designing control surfaces, wing mechanisms, and actuation systems. This method has many applications and has been considered for the design of CLD (constrained layer damping) behavior for vibration clampdown of an aircraft panel, for the design of an aerospace component in an F/A-18 aircraft bulkhead, and the control design of a transformable aircraft with wings with variable-sweep.

The third method considered is Bidirectional Evolutionary Structural Optimization, representing a significant advancement in topology optimization. This method allows for recovering elements that have been removed previously in the optimization process. It has been used in researchers' work as an optimization technique to maximize the volume or shear modulus in auxetic materials or negative Poisson's ratio, to improve the structural integrity by optimizing material distribution, and to ensure that critical load paths are efficiently supported. The rigidity of the support of an aircraft engine by achieving weight savings without compromising safety. The aerothermoelastic coupling for the wing of a hypersonic transport aircraft by mitigating thermal and aerodynamic effects during high-speed flight and optimizing a fixed wing on a single beam using an unstructured and irregular mesh, ensuring efficient load distribution and aerodynamic performance, among others.

The last method mentioned in the research is the Proportional Topology Optimization (PTO), is an effective and efficient tool for structural design, particularly when a rapid and straightforward optimization process is required. Although it may not always yield the most optimal solutions compared to more advanced methods, its simplicity and efficiency make it a valuable asset in structural optimization. With certain enhancements, PTO has been successfully applied to solve problems in mechanisms, beams, and linear elastic structures, demonstrating its applicability in the aeronautical field. In the context of morphing aircraft, PTO has been employed in various areas, including the design of morphing wing structures, the development of adaptive skin materials, and the optimization of load-bearing structures. The use of PTO allows researchers to balance critical objectives such as weight reduction, structural integrity, and aerodynamic performance, which are essential for advancing morphing aircraft technologies.

Based on the analysis presented in the Table 1, several aspects of topology optimization methods, including efficiency, handling of complex geometries, and mitigation of local redundancy, are discernible. The Solid Isotropic Material with Penalization (SIMP) method is computationally efficient due to its simple, penalty-based approach. In contrast, the Bidirectional Evolutionary Structural Optimization (BESO) method enhances efficiency by iteratively removing unnecessary material and introducing new material where needed. Both the SIMP and BESO methods exhibit superior capabilities in handling complex geometries, surpassing the performance of the Evolutionary Structural Optimization (ESO) method. SIMP adapts seamlessly to complex shapes in various aircraft

**Table 1.** Classification of topology optimization methods for morphing aircraft (source: created by authors)

| Method | Concept | Algorithm | Advantages | Challenges |
|---|---|---|---|---|
| Solid Isotropic Material with Penalization (SIMP) | The method optimizes the distribution of the material by penalizing intermediate densities. The objective is to discover the optimal structure of the material, minimalizing rigidity and subject to volume restrictions. | $E_e = E_{min} + x^p \left(E_0 - E_{min}\right);$ (1)<br>$E_e$ – effective Young's modulus.<br>$E_{\min}$ – minimum value for the elastic modulus.<br>$E_0$ – Young's modulus of the material.<br>$x$ – density variable (range 0 to 1).<br>$p$ – penalization factor. | Due to its simple penalty approach, it is computationally efficient and adjusts the material distribution simultaneously, allowing structural flexibility during the process. | This method still has problems with intricate designs or complex shapes, which cause localized material redundancy and affect overall efficiency. |
| Evolutionary Structural Optimization (ESO) | The optimization method has been inspired by natural selection, which seeks to eliminate regions of elements subjected to low stress and simplifies the design. | ▪ Start with a finite element model.<br>▪ Calculate element stresses.<br>▪ Remove elements with low stresses.<br>▪ Repeat until convergence. | It reduces the use of materials and avoids unnecessary complexity; it also allows aerodynamic loads to be modified smoothly, improving performance. | Has difficulty with intricate shapes and does not yet handle multidisciplinary aspects well, as it does not directly consider aerodynamic performance. |
| Bidirectional Evolutionary Structural Optimization (BESO) | Combine the ESO method with additional alternatives to remove and add material to achieve an optimal design. | ▪ Start with an initial design.<br>▪ Remove elements with low stresses (similar to ESO).<br>▪ Add material to regions with high stresses.<br>▪ Repeat until convergence. | Stabilities the addition and removal of material, resulting in better results than the ESO method. | It introduces additional complexity due to material addition steps and requires careful consideration. |
| Proportional Topology Optimization (PTO) | Determine the optimal material distribution within a given design space, under specific loads, boundary conditions, and constraints. Principally maximizing the performance of a structure stiffness (PTOs), compliance (PTOc) while minimizing the material usage. | PTOs<br>▪ Start with an initial material distribution (FE and stress analysis).<br>▪ Calculate the sensitivity of the compliance (respect changes in the material distribution).<br>▪ Update the material distribution proportionally to the sensitivity (more material or less material).<br>▪ Repeat until convergence.<br>PTOc<br>▪ Start with an initial guess of material distribution.<br>▪ Perform a sensitivity analysis considering the design.<br>▪ Proportional update of the material distribution based on the sensitivity analysis.<br>▪ Continue update the process. | Is relatively simple compared with the other methods, easier to implement and understand, suitable for large-scale problems, it is good for an initial design, serving as a preliminary step in a multi-stage optimization process. | It has limited flexibility in handling complex design constraints or multi-objective problems, requiring careful tuning for each specific problem. |

components, while BESO evolves structural layouts to accommodate intricate designs. Additionally, BESO effectively addresses local redundancy by interacting with the ESO and SIMP methods to optimize material distribution. In summary, SIMP offers computational efficiency, and BESO provides adaptability to complex geometries and local redundancy mitigation. The Proportional Topology Optimization (PTO) method offers a simpler and faster alternative by updating material distribution proportionally based on sensitivity analysis. Although efficient and easy to implement, PTO may not achieve the same level of detail as SIMP or BESO. Future research should explore the specific advantages of BESO and PTO and their applicability across various structural components in morphing aircraft.

## 4. Future of topology optimization

Morphing aircraft are designed to modify their aerodynamic design; for this reason, unique challenges are presented for the optimization of structural topology. In the future, these should be focused on flexible structures, flexible skins, control systems, complex geometries, manufacturing ability, and flight trajectory dependency, according Parancheerivilakkathil et al. (2024). Topology optimization is essential in designing efficient and adaptive morphing aircraft structures, mentioned in the research of Zhu et al. (2023). However, these challenges are detailed:

1. Computational cost: Iterative optimization algorithms may require high computational use to solve complex models. They require significant computa-

tional resources, especially for large-scale problems or problems with complex shapes, so accuracy and computational efficiency must be carefully balanced, discussed by Sun et al. (2012).

2. Mesh dependence: The topology optimization results are susceptible and dependent on the mesh resolution, which can lead to different optimal designs. Achieving mesh-independent solutions remains an ongoing challenge, requiring robustness studies and adaptive mesh refinement techniques.
3. Manufacturing Constraints: Manufacturing processes impose many practical limitations on design feasibility because topology-optimized structures can exhibit intricate or complex geometries that are difficult to manufacture. To ensure manufacturing capacity, manufacturing constraints must be considered and integrated into the optimization process.
4. Material interpolation: Morphing planes often involve multifunctional materials with nonlinear behaviour, so it is necessary to develop reliable material interpolation models to consider these complexities for successful topology optimization, as mentioned by Lampeas (2020).
5. Validation: Validating optimized designs through tests and trials is essential, according Criollo et al. (2022). Although it is based on simulation, it provides valuable information, but accompanied by experimental validation, it guarantees that the transformation structures meet the design requirements and constraints discussed by Parancheerivilakkathil et al. (2024).
6. Complex Geometries: Morphing wings require intricate internal structures, primarily to achieve shape changes. Dexl et al. (2020) noted that the topology optimization method must handle irregular shapes and complex material distributions, leaving traditional designs aside.
7. Manufacture: Many optimized structures are incompatible with conventional manufacturing methods. One solution is multi-material additive manufacturing, which offers a promising solution with customized materials and geometries to obtain more efficient and adaptable designs.
8. Flight Trajectory Dependency: Determining structural topology quickly for a morphing aircraft is a new challenge. The optimal topology depends on the specific flight path, creating a bidirectional relationship between aerodynamics and structural design, as mentioned by Rudnick-Cohen et al. (2023).
9. Achieving these challenges will lead to more efficient morphing aircraft designs that are adaptable to current needs and requirements.

## 5. Conclusions

This article provides an exhaustive review of various topology optimization techniques developed over time, with a primary focus on enhancing structures in morphing aircraft design. The evolution of topology optimization methods has profoundly influenced morphing aircraft design, leading to the development of structures that are both lightweight and adaptable to dynamic aerodynamic demands in main components (e.g., wings). Analysis of the literature reveals over 1,123 publications on this topic, with a notable increase in contributions, particularly from China, in recent years.

Among the methods reviewed, the Solid Isotropic Material with Penalization (SIMP) method is computationally efficient and adjusts the material distribution, thereby allowing structural flexibility. The Evolutionary Structural Optimization (ESO) method, inspired by natural selection, eliminates elements with low stress, and allows aerodynamic loads adjustments. The Bidirectional Evolutionary Structural Optimization (BESO) method enhances robustness by permitting the addition and removal of material, making it suitable for managing multiple loads and complex scenarios These previous methods have been extensively validated in morphing aircraft research, demonstrating their efficacy in this field. However, the Proportional Topology Optimization (PTO) method, despite its faster and versatility, remains underexplored in the context of morphing aircraft design and warrants further investigation.

Despite these advancements, the SIMP method is less effective for complex geometries, while the ESO method faces limitations with constraints beyond its original scope. The BESO method, though robust, demands significant computational resources, and the PTO method, while promising, is better suited for preliminary structural analysis.

The challenge for researchers is to develop novel Topology Optimization methods for morphing aircraft design, considering the computational cost, mesh dependency, manufacturing constraints, material interpolation, validation, complex geometries, manufacturing, and flight trajectory dependency.

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## Author contributions

Mena Carlos intellectualized the study, including the research objectives and data collection for analysis, and authored the initial manuscript draft. Criollo Linker created the data visualizations, conducted the literature review, and validated the results. Shen Xing analyzed and interpreted the information and supervised the review.

## Disclosure statement

The authors declare that no financial or professional rivalries, personal interests, or benefits have arisen, considering the direct applications of this research.

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