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MATHEMATICAL MODEL OF STABLE EQUILIBRIUM OPERATION OF THE FLIGHT SIMULATOR BASED ON THE STEWART PLATFORM

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Abstract. This paper analyses the available mathematical models of flight simulators based on the Stewart platform. It was found that there is no model that describes the conditions for stable dynamic equilibrium operation of the Stewart platform as a function of a number of important motion parameters. In this context, a new physical model is proposed based on classical models of theoretical mechanics using the d'Alembert formalism, the concept of stable equilibrium of a mechanical system. This model mathematically separates the stable equilibrium of the flight simulator motion system from the general uniformly accelerated motion. The systems of equations obtained in the framework of the model connect the physical and geometrical parameters of the Stewart platform and make it possible to determine the reactions in the upper hinges of the platform support, the limit values of the position angles in the space of the base of the support of the Stewart platform, under which the condition of stable equilibrium operation of the Stewart platform is satisfied. The proposed physical model and the analytical relations obtained on its basis are of great practical importance: the operator controlling the operation of the Stewart platform-based flight simulator can control the range of parameters during training so as not to bring the flight simulator out of stable equilibrium.

Keywords: algorithm of stable operation, limiting angles, equilibrium motion, Stewart platform, analytical solution.

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Introduction

A flight simulator is a device that artificially recreates the flight of an aircraft and its environment (Heintzman, 1996; Lapiska et al., 1993). In practice, a flight simulator is used for various purposes: pilot training, aircraft design and development, and the study of their characteristics, including stability and controllability. The Full Flight Simulator (International Civil Aviation Organization, 2015) Figure 1 includes an aircraft cabin and consists of several interconnected and interacting systems (flight dynamics, vision system, motion system, etc.).

The motion system is one of the most important components of the Full Flight Simulator. It conveys the feeling of being in a real, moving environment. To do this, the motion system must simulate the acceleration effects in all six degrees of freedom that the pilot experiences when moving freely in space. The modern motion system of full flight simulators usually uses a six degree of freedom motion system proposed by (Stewart, 1965) (Figure 1). With six degrees of freedom (three rotations along the pitch, roll and yaw axes and three translational movements along a vertical, transverse and longitudinal degree of freedom), the Stewart platform can perform a wide range of complex and combined movements within acceptable limits.

The six-stage Stewart platform is a spatial-kinematic system of six interconnected hydraulic actuators operating at angles of up to 70° relative to the vertical axis. Since the movements of the Stewart platform are generated by combining the movements of multiple hydraulic actuators, the Stewart platform is also referred to as synergistic because of the synergy (mutual interaction) between the programming of the hydraulic actuators. Because the Stewart platform has six hydraulic actuators, it is also referred to as a hexapod (six legs). The six-leg Stewart platform is a kinematically parallel system. With it, positioning errors in serial kinematic chains do not tend to propagate additively across the chain links, so it is able to perform positioning tasks with high accuracy. In addition, the parallel structure naturally distributes the forces/torques of the hydraulic actuators, resulting in high dynamic performance. The design is resistant to tilting, twisting around the vertical axis and other unacceptable movements of the Stewart platform. The disadvantage of this design is the mutual interference of the hydraulic actuators in redistributing the mass of the Stewart platform and the load acting on it to each hydraulic actuator.

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This significantly impairs operation and has a negative effect on the dynamic properties of the hydraulic drives. In order to eliminate this shortcoming, the power input of the hydraulic actuator and thus the power consumption must be significantly increased. The advantages of such a mechanism lie in the increased rigidity and compactness of the structure, the disadvantages in a possible loss of stability. The efficiency of a dynamic upright thus depends largely on the angular range of the mutual arrangement of the hydraulic actuators within which the stability of the robot is ensured. For the Stewart platform, the kinematic safety conditions must be met. The ratio between the sides of the upper and lower triangle as well as the length of the hydraulic actuators with the maximum extended and maximum retracted rod must be such that it is kinematically impossible to bring the Stewart platform into a dangerous position. The maximum available forces of the hydraulic actuators must also be sufficient to overcome the torques from the inertia forces of the Stewart platform in each of its positions (PPO "ERA", 1983).

The design of the flight simulator is such that the cabin is hinged to Stewart's moving platform. It is easy to see that such a mechanical linkage may be in a non-equilibrium state at certain angles of deviation of the hydraulic actuators from the stable initial state. This, in turn, can cause the entire structure of the simulator to tip on its side. For this reason, the question arises at what maximum permissible angles of deviation from the initial equilibrium state the Stewart platform can remain in the equilibrium state.

Purpose of the study. Due to the high cost of Stewart platforms and the increasing demands on the accuracy of motion cueing, the problem of determining limiting parameters of the Stewart platform under which stable equilibrium operation of the Stewart platform is possible is an urgent task. Therefore, the main objective of this study is to formulate and solve the dynamics problem: to establish the relationship between the responses of the upper hinges of the hydraulic actuators, the changing yaw, roll and pitch angles, and the maximum allowable angular deviations of the hydraulic actuators from the initial position of stable equilibrium.

1. State of arts

Currently, the problem of motion analysis and control of the work of robots with 6 degrees of freedom (DOF) is important in various branches of mechanical engineering. These include industrial robotic manipulators (Šegota et al., 2020), robots that mimic the motion of motor vehicles (Dymarek et al., 2014), and biorobots that simulate the motion of living beings (Huang et al., 2022). Robotic flight simulators, whose design is based on the Stuart platform with 6 degrees of freedom, play an important role in the training of pilots. Therefore, investigating the operation of the Stewart platform under different loading parameters is an important scientific and practical task.

The analysis of the publications on the simulation of the motion of the Stewart platform has shown that the existing works can be conditionally divided into the following areas: the study of the problem of direct and inverse kinematics of the Stewart platform; refinement of the geometry of the Stewart platform; analysis of the dynamics 6-DOF; development of numerical methods for solving nonlinear systems of kinematic equations of the 6-DOF; study of accuracy control, calibration, stability of the 6DOF operation.

The general theory of the Stewart platform is presented in the work of (Fichter, 1986). The direct kinematic solution and analysis of a Stewart platform is studied in the work of Nanua et al. (1989), Liu et al. (1993). The work of Petrescu et al. (2018) is devoted to the inverse problem of the motion kinematics of the Stewart platform. The dynamics of the Stewart platform has been studied in the works of Harib and Srinivasan (2003), Bingul and Karahan (2012), Lopes (2009), Leonov et al. (2014). Since the equations describing the operation of the Stewart platform are nonlinear, much attention is paid to the development of numerical algorithms to solve them. For example, the works of Nguyen et al. (1991), Yee and Lim (1997), Wang et al. (2009), Zhiyong et al. (2016), Sheng et al. (2006) and Dasgupta and Mruthyunjava (1996) provide algorithms for the numerical solution of the Stewart platform forward kinematics problem. The works of Dymarek et al. (2014), Yang et al. (2022) study the refined geometry problem for solving the direct 6DOF kinematics problem. Methods for Stewart platform accuracy control, sliding mode control and calibration are described in Zhuang et al. (1998), Chen and Fu (2013), Velasco et al. (2020), Ono et al. (2022) and Silva et al. (2022).

1.1. Use of the Stewart platform models in aviation

Motion cueing, which most closely resembles real flight conditions, is simulated by the Stewart platform (Figure 1). It consists of a simulator cabin rigidly connected to the base of the platform. The base in turn is pivotally attached to three pairs of hydraulic actuators that can simulate the spatial motion of the aircraft in six degrees of freedom.

Figure 1. Full flight simulator with the Stewart platform

Nowadays, all flight simulators are equipped with automatic devices containing the equations of motion of a free aircraft, including the kinematics and dynamics of the motion.

In Scholten et al. (2020) a model for linear control of changes in kinematic parameters was considered and a block diagram of in-flight parameter control was proposed. The works (Chandrasekaran et al., 2021; Scholten et al., 2020; Ahmed, 2012; Ahmad et al., 2021) are devoted to the issues of stable flight simulation. The authors (Chandrasekaran et al., 2021) developed a universal model of a flight simulator (Ground-based Variable Stability Flight Simulator) based on the optimization of a set of kinematic characteristics (angular velocity), aerodynamic force and moments. Different flight modes (Dutch-Mode, Roll-Mode), static longitudinal stability and control were investigated. In Scholten et al. (2020) a system for flight simulation with variable stability is presented. Two experiments were conducted: one experiment was performed on a stationary flight simulator, the second on a Cessna Citation II. The simulation experiment showed the difference between the full model including the actuator and the INDI (Incremental Nonlinear Dynamic Inversion) controller.

Longitudinal stability of a flight model using MATHLAB/SIMULINK was studied by (Ahmed, 2012), where the condition was verified. The evaluation of stability parameters for a wide-body aircraft using a computer was carried out by (Ahmad et al., 2021). In particular, the calculation of airfoil characteristics and aerodynamic coefficients was performed analytically and numerically, and the verification of stability conditions was an intermediate element in the calculation. A similar model for simulating a helicopter flight is presented in Pausder et al. (1992), in which the feedback loop is implemented based on a linear model whose variables are the components of velocity, angular velocity, and angular acceleration. In Das and Kumpas (2019), in addition to kinematic relations, the mathematical model of helicopter flight simulation uses force and torque equations that account for the inertial properties of the system. In Sapunov and Proshin (2011), a generalized simulation model of an *nth*-order dynamical system is proposed that describes the formation of control laws in a closed system by the motion of a vector of controlled coordinates, including components of velocity, acceleration, and overload. The above analysis of simulation on flight simulators (Stewart platforms) does not touch the issue of stable equilibrium operation of the Stewart platform itself as a mechanical system. In fact, at certain angles of the position of the lifting jacks, which are the basis for fixing the simulator, an unstable equilibrium condition and an unforeseen "blocking" of the Stewart platform on one of the sides may occur. Today there is a mathematical theory of stability motion offered by Lyapunov (Malkin, 1966; Barbashin, 1970). However, this theory studies only the local stability of the mathematical solution in the presence of small changes in the system parameters. In other words, in the language of modern mathematical analysis, Lyapunov theory examines the proposed solution, often without even finding it, for uniform convergence as a function of many variables. For example, in Andrijevskij et al. (2017) the direct and inverse problem of the Stewart platform operation is considered in terms of the change of lengths of hydraulic actuators, the equations of motion for forces and moments are written, and the moments of inertia of the system are considered. To verify the local stability of the motion of the mechanical system, the Lagrange-Dirichlet theorem was applied. Based on this system, the relationship between the feedback coefficient *G* and each parameter of the problem was determined. The feedback $G > 0{,}5867 mgz_c l^{-2}$ is introduced to control the operation of hydraulic cylinders. However, from the above inequality, it is clear that not all physical motion characteristics are included in the stability criterion for the Stewart platform motion. And curiously, the conditions for the equilibrium state of the platform are not directly stated in the paper. The second approach, the study of asymptotic stability based on Lyapunov's stability theory, allows to study only small deviations from the stable state (which, again, is not specified in the work in any way) of the lengths of the hydraulic actuators. And now what? But does such analysis give an answer to the question: what are the conditions of physical equilibrium of the platform depending on the roll, pitch and yaw angles? They are not included in any of the articles analyzed above.

Another approach to the study of the stability of Stewart platform is based on the analysis of the general equations of motion of a mechanical system, built on the equations of analytical mechanics and described in (Abramov & Mukharlamov, 2011). The essence of the proposed model is that real mechanical connections are replaced by program connections with disturbances. In this way, the influence on the operation of hydraulic actuators is studied and the parameters for their stable operation are selected. As in Malkin (1966), Barbashin (1970), the stability is considered asymptotically. However, nothing is said about the equilibrium stability of the mechanical system with six degrees of freedom as a whole, and the task is not even posed. As can be seen from the analysis of the above works, as well as from the available review articles by Markou et al. (2021), He et al. (2022), little attention has been paid to the issues of a stable equilibrium state of the operation of the Stewart platform mechanical system. Thus, in the work of Markou et al. (2021) only a simplified model of the equilibrium state is described: a static equilibrium state is considered. Equations for zero forces and torques relative to one of the hinge points are written (see Markou et al., 2021: Part 3. Equilibrium equations, equations 2–11). The authors of this paper write "...It should also be noted that the motion of the structure is very slow and therefore the behavior of the system is assumed to be static...". After that, only the conditions for a static equilibrium state are given: the principal torque and the principal vector of forces are equal to zero. However, the paper says nothing about how the problem is solved in the case of the dynamic behavior of a mechanical system. The paper does not provide specific correlations between the maximum allowable deflection angles of hydraulic actuators as a function of the specified weight of the platform and the yaw, roll, and pitch angles. It can be concluded that there is currently no dynamic model of the stable dynamic equilibrium of the Stewart platform that determines the limit values for the position angles of the hydraulic actuators at which the conditions for the equilibrium of the 6DOF platform are met.

At the same time, for practical purposes, it is important to know both the solution of the equilibrium motion problem of a mechanical system and the range of parameters that ensures the stability of the equilibrium of the entire six-degree-of-freedom mechanical system, not just the hydraulic actuators. In other words, the dynamic-physical stable equilibrium of the mechanical system with six degrees of freedom as a whole was under the scientific interest. Such a model has not existed before. The authors of this paper have posed and theoretically solved the problem of determining the limiting parameters of the stable equilibrium of Stewart platform of the 6DOF. A closed system of equations was established, which allows to determine the limiting angles for the position of the upper hydraulic actuators depending on the weight of the aircraft section, as well as the pitch, roll and yaw angles at which stable equilibrium operation of the Stewart platform is possible.

2. Model of stable operation of the Stewart platform. Quasi-static approach

2.1. Physical model of stable equilibrium

To understand the idea of the quasi-static approximation, the classical problem of theoretical mechanics is considered: the motion of a rigid body along a horizontal surface (Figure 2) under the action of a force F. It is known that the body is additionally acted upon by gravity, the force of the normal reaction of the support, and the force of Coulomb friction. If the Cartesian coordinate system is introduced, the vertical projections of the gravity force and the normal force cancel each other out. The situation is quite different in the horizontal direction of motion: to start the motion, a minimum force is required to move the body from its place. In mechanics, this force is called the static frictional force, the overcoming of which in this case sets the body

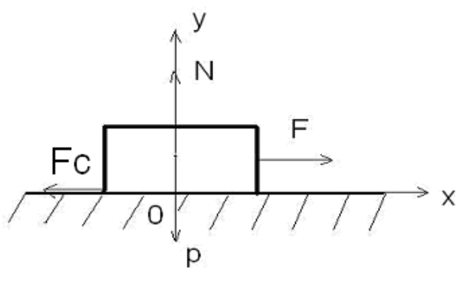

Figure 2. To the explanation of the body motion stability

in a rectilinear translational motion. When the force acting on the body exceeds the static frictional force, this results in acceleration. It is known from theoretical mechanics that the stable behavior of the body is related only to the compensation of the force when it is close to the resistance force, in this case the Coulomb friction force.

The above example leads to follow the idea. The question of the stable state of equilibrium of the Stewart platform must be connected with those limit values of forces and torques which correspond to the reactions of the hydraulic actuators and reactive torques. If the active forces in the system exceed the reactive forces, the Stewart platform may move with acceleration. In this case, the excess component of the active forces $\triangle\vec{F}$ is used to produce accelerated motion:

$$\triangle\vec{F} = \vec{F} - \vec{F}_c = m\vec{a}\,. \tag{1}$$

In this work we will not study the component, since we are interested only in those limit values of the forces in hydraulic actuators for which the condition of the equilibrium state of the Stewart platform is satisfied. And these limit values correspond to certain values of roll, pitch and yaw angles as well as weight of the simulator space. It follows that the active forces in the upper joints of each of the three pairs of hydraulic actuators can be conditionally divided into two components: the first is responsible for the reaction $\vec{R}_1, \vec{R}_2, \vec{R}_3$ of the mechanical system, the second $\triangle\vec{F}_1, \triangle\vec{F}_2, \triangle\vec{F}_3$ for the accelerated motion:

$$\vec{F}_1 = \vec{R}_1 + \triangle\vec{F}_1, \vec{F}_2 = \vec{R}_2 + \triangle\vec{F}_2, \vec{F}_3 = \vec{R}_3 + \triangle\vec{F}_3\,. \tag{2}$$

Based on Eq. (2), Newton's the 2nd law can be formulated in the following way:

$$\vec{R}_1 + \triangle\vec{F}_1 + \vec{R}_2 + \triangle\vec{F}_2 + \vec{R}_3 + \triangle\vec{F}_3 + \vec{P} = m\vec{a}, \tag{3}$$

where $\vec{P}$, $\vec{a}$ is the weight of the simulator cabin together with the base of its attachment and the acceleration of the system center of mass. From the above:

$$\vec{F}_1 + \triangle\vec{F}_2 + \triangle\vec{F}_3 = m\vec{a}\,,$$
$$\vec{R}_1 + \vec{R}_2 + \vec{R}_3 + \vec{P} = \vec{0}\,. \tag{4}$$

As for the torque generated in the system, the reactions $\vec{R}_1, \vec{R}_2, \vec{R}_3$ together with gravity generate mutually balancing torques, and the increments of the forces $\triangle\vec{F}_1, \triangle\vec{F}_2, \triangle\vec{F}_3$ generate torque. Together they form the main vector of the torque $\vec{M}$:

$$\vec{M}_{\vec{R}} + \vec{M}_{\triangle\vec{F}} + \vec{M}_{\vec{P}} = \vec{M}\,, \tag{5}$$

where

$$\vec{R} = \vec{R}_1 + \vec{R}_2 + \vec{R}_3,$$

$$\triangle\vec{F} = \triangle\vec{F}_1 + \triangle\vec{F}_2 + \triangle\vec{F}_3\,.$$

Since the conditions of a stable equilibrium are under interest, it follows from Eq. (5):

$$\vec{M}_{\vec{R}} + \vec{M}_{\vec{P}} = \vec{0}\,. \tag{6}$$

If the Stewart platform moves with infinitesimal acceleration, then such motion occurs at nearly constant velocity. And the system of Equations (4)–(6) is indeed a quasi-static condition for a stable equilibrium state. If conditions (Equations (4)–(6)) are satisfied, the Stewart platform will not tip over because the overturning moment and the resulting forces are zero.

2.2. Projection of forces and moments

In the search for a concrete mathematical implementation of the physical model proposed above, it has become apparent that the earth normal and the associated coordinate systems are the most suitable for carrying out the projection onto the axes that will give the desired result. It is assumed that each of the two hydraulic actuators (Figure 3a, Kompleksny trenazher; Simulator, n.d.) belonging to one of the three pairs moves symmetrically, so that the guides of the hydraulic actuators of a pair form the sides of an isosceles triangle (Figure 3b). This means that the total reactions $\vec{R}_i, i = \overline{1,3}$ in the upper hydraulic actuator joints are directed along the heights of the isosceles triangles drawn to the bases (the segments between the lower pairs of joints).

The reactions $\vec{R}_i$ are located at a certain angle with respect to the mounting plane of the simulator cabin base. It is assumed that the angles between the corresponding reactions and the heights of an equilateral triangle, which is the simulator booth mounting base, are as follows:

$$\angle(\vec{R}_1, h_a) = \alpha, \angle(\vec{R}_2, h_b) = \beta, \angle(\vec{R}_3, h_c) = \gamma. \quad (7)$$

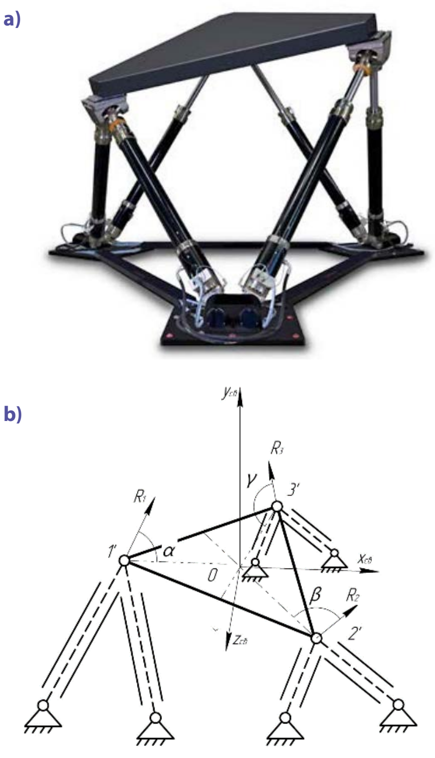

Figure 3. Stewart platform in operation: (a) system of 6 movable hydraulic actuators; (b) block diagram

Without loss of generality, it is assumed that during operation vertex *1* is in the upper position and vertices *2* and *3* are in the lower position with respect to the initial horizontal position of the mounting base of the hydraulic actuator chamber (Figure 3b). Such an assumption is necessary for the uniqueness of the projection onto the axes of the associated coordinate system. Since all three vertices are equal, the triangle used to attach the simulator chamber is equilateral; the symbolic designation of the vertices does not impose any additional constraints on the solution of the problem.

The origin of the associated coordinate system $OX_1Y_1Z_1$ in the center of the circle circumscribed around an equilateral triangle – the base of the simulator chamber attachment is chosen. Then, taking into account the projection on the axis, was gained the following:

$$\vec{R}_{1acs} = \left\{R_1 \cos\alpha \cos 30^\circ; R_1 \sin\alpha; -R_1 \cos\alpha \sin 30^\circ\right\} = \left\{\frac{\sqrt{3}}{2} R_1 \cos\alpha; R_1 \sin\alpha; -\frac{1}{2} R_1 \cos\alpha\right\},$$

$$\vec{R}_{2acs} = \left\{-R_2 \cos\beta \cos 30^\circ; R_2 \sin\beta; -R_2 \cos\beta \sin 30^\circ\right\} = \left\{-\frac{\sqrt{3}}{2} R_2 \cos\beta; R_2 \sin\beta; -\frac{1}{2} R_2 \cos\beta\right\},$$

$$\vec{R}_{3acs} = \left\{0; R_3 \sin\gamma; R_2 \cos\gamma\right\}. \quad (8)$$

Much more important in force projection is the choice of a coordinate system for projecting the gravity of the compartment. If the gravity is projected of the compartment onto the axes of the normal Earth coordinate system, then the only one component of the projection onto the vertical axis is obtained. This does not allow to establish a mathematical connection between the unknowns that must be determined in solving the problem. Therefore, in order to obtain mathematical dependencies related to the parameters of the physical system, it is useful to project gravity onto the axes of the associated coordinate system. It is known (Gorbatenko et al., 1969) that the transition matrix between the normal Earth coordinate system and the bound coordinate system has the following form:

$$A_{33} = \begin{bmatrix} \cos\vartheta\cos\psi & \sin\upsilon & -\cos\upsilon\sin\psi \\ -\cos\gamma\sin\vartheta\cos\psi + \sin\gamma\sin\psi & \cos\vartheta\cos\gamma & \cos\gamma\sin\vartheta\sin\psi + \sin\gamma\cos\psi \\ \sin\gamma\sin\upsilon\cos\psi + \cos\gamma\sin\psi & -\sin\gamma\cos\vartheta & -\sin\psi\sin\vartheta\sin\gamma + \cos\psi\cos\gamma \end{bmatrix}, \quad (9)$$

where roll, pitch, and yaw angles are corresponding γ, ψ, ϑ. The relationship between gravity components in normal earth and bound coordinate systems is as follows:

$$\vec{P} = A_{33}\vec{P}_{acs}, \quad (10)$$

or in the form of a matrix:

$$\begin{bmatrix} 0 \\ -P \\ 0 \end{bmatrix} = A_{33} \begin{bmatrix} P_{xacs} \\ P_{yacs} \\ P_{zacs} \end{bmatrix}. \quad (11)$$

After multiplication, the following equations are obtained to determine the unknown components $P_{xacs}, P_{yacs}, P_{zacs}$:

$$\begin{aligned}&P_{xacs}\cos\upsilon\cos\psi+P_{yacs}\sin\upsilon-P_{zacs}\cos\upsilon\sin\psi=0;\\&P_{xacs}(-\cos\gamma\cos\psi\sin\upsilon+\sin\gamma\sin\psi)+P_{yacs}\cos\gamma\cos\upsilon+\\&P_{zacs}(\cos\gamma\sin\upsilon\sin\psi+\sin\gamma\cos\psi)=-P;\\&P_{xasc}(\sin\gamma\cos\psi\sin\upsilon+\cos\gamma\sin\psi)-P_{yasc}\sin\gamma\cos\upsilon+\\&P_{zasc}(-\sin\gamma\sin\upsilon\sin\psi+\cos\gamma\cos\psi)=0.\end{aligned}\qquad(12)$$

The system of Equations (12) using Cramer's rule are solved. In order to do this, you must first calculate the determinant of the matrix A_{33}:

$$\begin{aligned}&\det A_{33}=\cos^2\upsilon\cos\psi\cos\gamma\cdot(-\sin\psi\sin\gamma\sin\upsilon+\\&\cos\psi\cos\gamma)+\cos^2\upsilon\sin\psi\sin\gamma\times(-\cos\gamma\sin\upsilon\cos\psi+\\&\sin\gamma\sin\psi)+\sin\upsilon(\cos\gamma\sin\upsilon\sin\psi+\sin\gamma\cos\psi)(\sin\gamma\sin\upsilon\times\\&\cos\psi+\cos\gamma\sin\psi)+\cos^2\upsilon\sin\psi\cos\gamma\cdot(\sin\upsilon\sin\gamma\cos\psi+\\&\cos\gamma\sin\psi)-\sin\gamma\cos\psi\cos^2\upsilon\sin\psi-\sin\vartheta\cdot(\cos\gamma\sin\upsilon\cos\psi+\\&\sin\gamma\sin\psi)(-\sin\psi\sin\gamma\sin\upsilon+\cos\psi\cos\gamma)=-\cos^2\upsilon\cos\psi\times\\&\cos\gamma\sin\psi\sin\gamma\sin\upsilon+\cos^2\upsilon\cos^2\psi\cos^2\gamma-\cos^2\upsilon\sin\psi\sin\gamma\times\\&\cos\psi\cos\gamma+\cos^2\upsilon\sin^2\psi\sin^2\gamma+\sin^3\upsilon\cos\gamma\sin\gamma\sin\psi\cos\psi+\\&\sin^2\upsilon\cos^2\gamma\sin^2\psi+\sin^2\gamma\sin^2\upsilon\cos^2\psi+\sin\upsilon\sin\psi\sin\gamma\cos\psi\cos\gamma+\\&\cos^2\upsilon\cos\gamma\sin\gamma\sin\psi\sin\upsilon\cos\psi-\sin\gamma\cos^2\upsilon\cos\psi\sin\psi-\\&\sin^3\upsilon\cos^2\gamma\cos\psi\sin\psi=\\&-2\cos^2\upsilon\cos\psi\cos\gamma\sin\psi\sin\gamma\sin\upsilon+\cos^2\upsilon\cos^2\gamma+\sin^2\psi\sin^2\gamma+\\&\sin^2\upsilon\cos^2\gamma+\sin^3\upsilon\cos\gamma\sin\gamma\sin\psi\cos\psi+\sin\upsilon\sin\psi\sin\gamma\cos\psi\cos\gamma+\\&\cos^2\upsilon\cos\gamma\sin\gamma\sin\psi\sin\upsilon\cos\psi-\sin\gamma\cos^2\upsilon\cos\psi\sin\psi-\\&\sin^3\upsilon\cos\gamma\cos\psi\sin\psi+\sin^2\gamma\sin^2\upsilon\cos^2\psi.\end{aligned}\qquad(13)$$

Using the d'Alembert formalism, it is assumed that the fictitious inertial forces on the upper hinges of the cabin to keep the body balanced are exerted. The total effect of the forces on the upper hinges can be described as follows:

$$\vec{F}=\vec{R}+m\vec{a}.\qquad(14)$$

Thus, using the kinetostatic method, the force load on the upper hinges into the reactions of the bonds and the inertial force is divided. The inertial forces are responsible for the accelerated motion of the body, while the reactions of the hinges are responsible for its stability: if it is assumed that the acceleration is negligible, then condition (Eq. (13)) actually turns into a quasi-static equilibrium condition for the base of the simulator cabin attachment. Based on Cramer's rule for determining $P_{x_{acs}}, P_{y_{acs}}, P_{z_{acs}}$, the following relationships are obtained:

$$P_{x_{acs}}=\frac{\det A_1}{\det A_{33}},P_{y_{acs}}=\frac{\det A_2}{\det A_{33}},P_{z_{acs}}=\frac{\det A_3}{\det A_{33}}.\qquad(15)$$

The necessary calculations are made:

$$\det A_1=\begin{vmatrix}0&\sin\upsilon&-\cos\upsilon\sin\psi\\-P&\cos\vartheta\cos\gamma&\cos\gamma\sin\vartheta\sin\psi+\sin\gamma\cos\psi\\0&-\sin\gamma\cos\vartheta&-\sin\psi\sin\vartheta\sin\gamma+\cos\psi\cos\gamma\end{vmatrix}=$$

$$-P\sin\gamma\cos^2\vartheta\sin\psi+P\sin\upsilon\cos\psi\cos\gamma-P\sin^2\vartheta\sin\psi\sin\gamma.$$

$$\det A_2=\begin{vmatrix}\cos\vartheta\cos\psi&0&-\cos\upsilon\sin\psi\\-\cos\gamma\sin\vartheta\cos\psi+\sin\gamma\sin\psi&-P&\cos\gamma\sin\vartheta\sin\psi+\sin\gamma\cos\psi\\\sin\gamma\sin\upsilon\cos\psi+\cos\gamma\sin\psi&0&-\sin\psi\sin\vartheta\sin\gamma+\cos\psi\cos\gamma\end{vmatrix}=$$

$$\begin{aligned}&-P\sin\gamma\cos^2\vartheta\sin\psi+P\sin\upsilon\cos\psi\cos\gamma-P\sin^2\vartheta\sin\psi\sin\gamma\\&-P\cos\upsilon\cos\psi(-\sin\psi\sin\upsilon\sin\gamma+\cos\psi\cos\gamma)-\\&P\cos\upsilon\sin\psi(\sin\upsilon\sin\gamma\cos\psi+\cos\gamma\sin\psi)=\\&-P\cos\upsilon\cos\gamma(\cos^2\psi+\sin^2\psi)=-P\cos\upsilon\cos\gamma\end{aligned}$$

$$\det A_3=\begin{vmatrix}\cos\vartheta\cos\psi&\sin\upsilon&0\\-\cos\gamma\sin\vartheta\cos\psi+\sin\gamma\sin\psi&\cos\vartheta\cos\gamma&-P\\\sin\gamma\sin\upsilon\cos\psi+\cos\gamma\sin\psi&-\sin\gamma\cos\vartheta&0\end{vmatrix}=$$

$$\begin{aligned}&-P\sin\upsilon(\cos\psi\sin\upsilon\sin\gamma+\sin\psi\cos\gamma)-P\sin\gamma\cos\upsilon\times\\&(-\cos\gamma\sin\upsilon\cos\psi+\sin\gamma\sin\psi)=-P\sin^2\upsilon\cos\psi s\in\gamma-\\&P\sin\upsilon\sin\psi\cos\gamma+P\sin\upsilon\cos\psi\sin\gamma\cos\upsilon\cos\gamma-P\sin^2\gamma\cos\upsilon s\in\psi.\end{aligned}\qquad(16)$$

After these determinants are calculated, $P_{x_{acs}}, P_{y_{acs}}, P_{z_{acs}}$ can calculate with the Eq. (15).

3. State of the balance of forces

Since the conditions for the equilibrium of forces are invariant with respect to the choice of coordinate system, having found the projections of the components of gravity on the axes of the associated coordinate system and knowing the projections of the reactions in the upper joints, it is possible to write the conditions for the equilibrium of forces in the following form:

$$\begin{aligned}&R_{1xasc}+R_{2xasc}+R_{3ascx}+P_{xasc}=0,\\&R_{1xasy}+R_{2ascy}+R_{3ascy}+P_{yasc}=0,\\&R_{1ascz}+R_{2ascz}+R_{3ascz}+P_{zasc}=0.\end{aligned}\qquad(17)$$

Considering the relation (Eq. (8)), the system of Eq. (17) can be written as follows:

$$\begin{aligned}&\frac{\sqrt{3}}{2}R_1\cos\alpha+\frac{\sqrt{3}}{2}R_2\cos\beta+0=-P_{xasc},\\&R_1\sin\alpha+R_2\sin\beta+R_3\sin\gamma=-P_{yasc},\\&-\frac{1}{2}R_1\cos\alpha-\frac{1}{2}R_2\cos\beta+R_3\cos\gamma=-P_{zasc}\end{aligned}\qquad(18)$$

After substituting in $P_{xasc}, P_{yasc}, P_{zasc}$ (Eq. (18)), the unknowns R_1, R_2, R_3 remain in this system of equations. They can also be calculated using Cramer's rule:

$$R_1=\frac{\begin{vmatrix}-P_{x_{asc}}&\frac{\sqrt{3}}{2}\cos\beta&0\\-P_{yasc}&\sin\beta&\sin\gamma\\-P_{zasc}&-\frac{1}{2}\cos\beta&\cos\gamma\end{vmatrix}}{\begin{vmatrix}\frac{\sqrt{3}}{2}\cos\alpha&\frac{\sqrt{3}}{2}\cos\beta&0\\\sin\alpha&\sin\beta&\sin\gamma\\-\frac{1}{2}\cos\alpha&-\frac{1}{2}\cos\beta&\cos\gamma\end{vmatrix}},$$

$$R_2=\frac{\begin{vmatrix}\frac{\sqrt{3}}{2}\cos\alpha&-P_{xasc}&0\\\sin\alpha&-P_{yasc}&\sin\gamma\\-\frac{1}{2}\cos\alpha&-P_{zasc}&\cos\gamma\end{vmatrix}}{\begin{vmatrix}\frac{\sqrt{3}}{2}\cos\alpha&\frac{\sqrt{3}}{2}\cos\beta&0\\\sin\alpha&\sin\beta&\sin\gamma\\-\frac{1}{2}\cos\alpha&-\frac{1}{2}\cos\beta&\cos\gamma\end{vmatrix}},$$

$$R_3=\frac{\begin{vmatrix}\frac{\sqrt{3}}{2}\cos\alpha&\frac{\sqrt{3}}{2}\cos\beta&-P_{xasc}\\\sin\alpha&\sin\beta&-P_{yasc}\\-\frac{1}{2}\cos\alpha&-\frac{1}{2}\cos\beta&-P_{zasc}\end{vmatrix}}{\begin{vmatrix}\frac{\sqrt{3}}{2}\cos\alpha&\frac{\sqrt{3}}{2}\cos\beta&0\\\sin\alpha&\sin\beta&\sin\gamma\\-\frac{1}{2}\cos\alpha&-\frac{1}{2}\cos\beta&\cos\gamma\end{vmatrix}}.\qquad(19)$$

4. Torque equilibrium condition

The torque equilibrium condition must be satisfied to ensure stable equilibrium of the simulator cabin with respect to the upper hydraulic drive joints. An additional condition for the equilibrium condition of the mechanical system, besides the equality of the principal force vector, is the equality of the principal torque of the forces acting on the simulator cabin and the upper hinges. With this condition, the tilt stability of the platform is actually guaranteed. It should be noted that in free flight, the fulfillment of these conditions is required only there is no fastening of the simulator cabin hinges, i.e. additional connections. This problem is different from the problem of stability of free flight from the point of view of theoretical mechanics. Since the center of the associated coordinate system is usually chosen in the center of an equilateral triangle – the base of the attachment of the upper hydraulic drive hinges – the center of gravity of the simulator cabin is located in the center of this triangle. The moment relation in the bound coordinate system has the form:

$$\sum M(0;0;0)_{acs} = \vec{R}_{1acs} \times \begin{bmatrix} x_{v_1} \\ y_{v_1} \\ z_{v_1} \end{bmatrix} + \vec{R}_{2acs} \times \begin{bmatrix} x_{v_2} \\ y_{v_2} \\ z_{v_2} \end{bmatrix} + \vec{R}_{3acs} \times \begin{bmatrix} x_{v_3} \\ y_{v_3} \\ z_{v_3} \end{bmatrix} + \begin{bmatrix} P_{x_{acs}} \\ P_{y_{acs}} \\ P_{z_{acs}} \end{bmatrix} \times \begin{bmatrix} x_{acs} \\ y_{acs} \\ z_{acs} \end{bmatrix} = \vec{0}. \quad (20)$$

Since the point at which gravity acts coincides with the origin of the associated coordinate system, the last term on the left-hand side of Eq. (20) is equal to the zero vector: $x_{asc} = 0, y_{asc} = 0, z_{asc} = 0$. Therefore, taking into account the relation (Eq. (8)), it is obtained:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\sqrt{3}}{2} R_1 \cos\alpha & R_1 \sin\alpha & -\frac{1}{2} R_1 \cos\alpha \\ x_{v_1} & y_{v_1} & z_{v_1} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{\sqrt{3}}{2} R_2 \cos\beta & R_2 \sin\beta & -\frac{1}{2} R_2 \cos\beta \\ x_{v_2} & y_{v_2} & z_{v_2} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & R_3 \sin\gamma & R_3 \cos\gamma \\ x_{v_3} & y_{v_3} & z_{v_3} \end{vmatrix} = \vec{0}. \quad (21)$$

After calculating the determinants the terms for each of the basis $\vec{i}, \vec{j}, \vec{k}$ are grouped:

$$\vec{i} \cdot (R_1 \sin\alpha z_{v_1} + \frac{1}{2} R_1 \cos\alpha y_{v_1} + R_2 \sin\beta z_{v_2} + \frac{1}{2} R_2 \cos\beta y_{v_2} + R_3 \sin\gamma z_{v_3} - R_3 \cos\gamma y_{v_3}) - \vec{j} \cdot (\frac{\sqrt{3}}{2} R_1 \cos\alpha z_{v_1} + \frac{1}{2} R_1 \cos\alpha x_{v_1} - \frac{\sqrt{3}}{2} R_2 \cos\beta z_{v_2} + \frac{1}{2} R_2 \cos\beta x_{v_2} - R_3 \cos\gamma x_{v_3}) + \vec{k} \cdot (\frac{\sqrt{3}}{2} R_1 \cos\alpha y_{v_1} - R_1 \sin\alpha x_{v_1} - \frac{\sqrt{3}}{2} R_2 \cos\beta y_{v_2} - R_2 \sin\beta x_{v_2} - R_3 \sin\gamma x_{v_3}) = \{0;0;0\}. \quad (22)$$

Each vector component on the left side of Eq. (22) must be equal to 0, since the right side is a zero vector. This results in the following system of equations:

$$\begin{aligned} &\frac{\sqrt{3}}{2} R_1 \cos\alpha y_{v_1} - R_1 \sin\alpha x_{v_1} - \frac{\sqrt{3}}{2} R_2 \cos\beta y_{v_2} - R_2 \sin\beta x_{v_2} - R_3 \sin\gamma x_{v_3} = 0, \\ &\frac{\sqrt{3}}{2} R_1 \cos\alpha z_{v_1} + \frac{1}{2} R_1 \cos\alpha x_{v_1} - \frac{\sqrt{3}}{2} R_2 \cos\beta z_{v_2} + \frac{1}{2} R_2 \cos\beta x_{v_2} - R_3 \cos\gamma x_{v_3} = 0, \\ &\frac{\sqrt{3}}{2} R_1 \cos\alpha y_{v_1} - R_1 \sin\alpha x_{v_1} - \frac{\sqrt{3}}{2} R_2 \cos\beta y_{v_2} - R_2 \sin\beta x_{v_2} - R_3 \sin\gamma x_{v_3} = 0. \end{aligned} \quad (23)$$

After substituting in (Eq. (23)) R_1, R_2, R_3, calculated on the basis of the Eq. (19), this system of equations becomes an essentially non-linear system of transcendental equations with respect to the desired angles α, β, γ. Such systems of equations only allow for a numerical solution. By numerically solving the system of Eq. (23), the limit values or critical values of the angles are obtained: an increase of at least one of the angles α, β, γ beyond the numerically determined limit values leads to an unstable state of equilibrium. The range of stable equilibrium operation of the Stewart platform is thus in the following range:

$$\alpha_{kp} < \alpha < \pi/2; \pi/2 < \beta < \beta_{kp}; \pi/2 < \gamma < \gamma_{kp}. \quad (24)$$

The conditions of fulfillment (Eq. (24)) mean the following. The operator controlling the movement of the flight simulator must monitor the magnitude of the forces in the hydraulic actuators as they compensate for the reactions in the upper hinges to maintain a stable equilibrium. In practice, it is necessary to create a computer program that contains all the analytical mathematical relationships described above in order to operate the Stewart platform. This program should calculate the maximum allowable values of the angles according to the given values of the input parameters (weight, roll, yaw and pitch angles) to meet the conditions for stable equilibrium. If the given values of the input parameters of the simulator do not allow to fulfill the conditions for stable equilibrium, the program (the brain of the robot) should issue the following message to the operator: "The parameters you have given bring the dynamic system out of the state of stable equilibrium. Please specify a different range of control parameters".

5. Algorithm for checking the stable equilibrium position of the platform

The iterations of algorithm are:

1. Determine the operating parameters of the platform: weight, pitch, roll and yaw angles.
2. Determine the absolute values R_1, R_2, R_3 of the reactions in the upper hydraulic drive joints.
3. Using the values of the reactions R_1, R_2, R_3, the projections $P_{x_{acs}}, P_{y_{acs}}, P_{z_{acs}}$ of the simulator compartment weight on the axis of the associated coordinate system, determine the limit values of the angles α, β, γ between

the modules of the normal reactions and the corresponding heights of an equilateral triangle to which the hydraulic actuators are symmetrically attached.

Conclusions

1. In this paper, a new analytical physical model for the stable dynamic equilibrium operation of the Stewart platform is proposed. This model application derived a closed system of equations that describe the stable equilibrium state of the Stewart platform, linking the physical and geometrical parameters of the mechanical system.
2. In the context of kinetostatics, the general motion is divided into the uniformly accelerated motion of the center of inertia and the equilibrium stable motion of the Stewart platform.
3. An analytical dependence of the suspension reactions of the upper hinges on the weight of the simulator cabin and on the pitch, roll and yaw angles was determined. For them, the limiting angles for the position of the base of the simulator cabin mounting are determined at which stable equilibrium is achieved.
4. Fulfillment of the conditions of the offered model made it possible to determine the values of the reactions R_1, R_2, R_3 in the upper hinges of the platform attachment. These values correspond to the limiting angles α, β, γ.
5. The new physical model proposed in this work and the analytical relations obtained on its basis are of great practical importance. The operator can use a programmed automatic to control the operation of the flight simulator based on the Stewart platform to ensure that the parameters set during training do not throw the flight simulator out of stable equilibrium.

Contributions of the authors

Kabanyachiy V. V. wrote an introduction and described the general state of the problem of motion simulation on dynamic stands. In particular, the problem of describing the stability of the operation of the Stewart platform is presented as unsolved so far.

Lukianov P. V. carried out an analysis of the available mathematical models for the operation of Stewart platforms, which in fact showed the absence of conditions for the stable operation of the Stewart platform as a whole as a mechanical system. In this context, Lukianov P. V. proposed a new physical model for the stable equilibrium operation of the Stewart platform, implemented in the form of a closed system of equations, which makes it possible to determine the range of limiting parameters of the stable equilibrium of the Stewart platform. Based on this model, an algorithm for controlling a state of equilibrium was developed. This algorithm allows monitoring the choice of the range given by the operating parameters of the flight simulator based on the Stewart platform.

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Notations

Symbols and abbreviations

$\vec{a}$ is the acceleration of the center of mass of the system;

$dC_m / d\alpha < 0$ is the condition of longitudinal stability (see Ahmed, 2012);

det A is the determinant of matrix A;

F is the Coulomb force acting on the body;

F_c is the frictional force;

g is the acceleration due to gravity;

G is the feedback coefficient;

h_a, h_b, h_c are the heights of the triangle ΔABC;

m is the body weight;

$\vec{M}_{\vec{R}} + \vec{M}_{\triangle\vec{F}} + \vec{M}_{\vec{P}} = \vec{M}$ is a total moment of the forces $\vec{R}, \triangle\vec{F}, \vec{P}$;

O_{xy} is the Cartesian coordinate system;

$\vec{P}$ is the weight of the simulator cabin together with the base of its attachment;

$\vec{P}_{asc} = \left\{P_{x_{asc}}, P_{y_{asc}}, P_{z_{asc}}\right\}$ is the weight of the simulator cabin together with the base of its attachment in the associated system of coordinates;

$\vec{R}_1, \vec{R}_2, \vec{R}_3$ are the reaction forces in each of the upper hinges;

$\vec{R}_{1acs}, \vec{R}_{2acs}, \vec{R}_{3acs}$ are the reaction forces in each of the upper hinges in the associated system of coordinates;

α, β, γ are angles between the heights of the base of the triangle and the reactions;

γ, ψ, ϑ are the roll, pitch, yaw angles;

$\triangle\vec{F} = \triangle\vec{F}_1 + \triangle\vec{F}_2 + \triangle\vec{F}_3$ is the active force producing accelerated motion in each of the upper hinges.