

NUMERICAL MODELLING OF TURBOFAN ENGINE DETERIORATION AS A FACTOR IN THE AIRLINES FUEL CONSERVATION

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Abstract. The boosting of the fuel efficiency of in-service aircraft is an issue of great commercial and ecological importance. One of the ways to achieve this is by adjusting the flight parameters and flight planning to the particular performance level of every single airplane. Main contributors to the aircraft performance deterioration are the aerodynamic and power plant deterioration. In this paper a mathematical modelling approach for the estimation of the effect of turbofan engine deterioration on passenger aircraft performance is proposed. Based on previous flight models developed by the authors, the present model simulates the deterioration of CFM-56-like turbofans on an Airbus A319-like airplane, and makes possible to compare the performance of airplanes with deteriorated and not deteriorated engines over various flight missions. A representative scenario is explored as an illustration. The model can be further developed to include the aerodynamic deterioration of the aircraft as well as other operational factors.

Keywords: aerospace simulation, aircraft propulsion, fuel conservation, turbofan engine deterioration, aircraft performance.

Introduction

Recently the potential of minimizing the direct and indirect operational costs as well as the environmental impact by optimizing the operations and maintenance has attracted a large interest among the airline industry. The main mover of this interest is the development of information technologies that enable almost real-time data acquisition and analysis of a wide spectrum of parameters ("Big data"), which was impossible a few years ago. An indicative study in this direction was conducted by Zou et al. (2014), estimating that the average airline fuel efficiency in the US airline industry for the year 2010 was 9-20% less than that of the most efficient carrier. The aircraft manufacturers also put efforts to assist their customers to raise the fuel efficiency of in-service aircraft and they propose a broad range of measures in this direction (Airbus, 2008; Roberson, 2010). The issue is also identified by International Air Transport Association (IATA, n.d.), which developed guideline materials and is offering training courses on fuel efficiency. Furthermore there is also a boom in the developed of specialized airline fuel efficiency software products by renown aviation industry players as well as by start-ups. The claim of IATA is that the airlines with a structured internal fuel efficiency program in place can achieve average additional fuel savings of 2–6% of fuel budget.

The complexity of the problem can be felt from multiple publications. Singh, Sharma, and Vaibhav (2012) make effort to identify the dimensions of the fuel consumption optimization, based on a literature review, the publication can serve as a basis for further studies. Marques and Leal (2012) propose a methodology for quantifying the fuel savings of airlines. There are two main problems outlined by the authors. First one is the aviation data quality (data management) of the numerous sources used. The same problem is profoundly discussed by Keller (2016). The second problem outlined in Keller's article (2016), is about the evaluation basis for fuel efficiency evaluation of aircraft operations. The need to look at various aggregation levels to properly compare the fuel consumption is stated.

The effort in the present work is aimed at the second of the problems stated above. The objective is to evaluate the potential for fuel saving by optimizing the flight operations with regard to the actual level of deterioration of every engine of the airplane. A simulation-based approach is used. Based on previous flight models developed by the authors,

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This is an Open Access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited. the present model simulates the deterioration of CFM-56-like turbofans on an Airbus A319-like airplane. The estimation of the operational fuel saving is performed by comparing the performance of airplanes with deteriorated and not deteriorated engines over typical flight missions.

The gas turbine engine deterioration problem itself is well studied in works like Meher-Homji, Chaker, and Motiwala (2001), Zaita et al. (1997), Verbist et al. (2011) and Lakshminarasimha, Boyce, and Meher-Homji (1992), but mostly the reliability and maintenance aspects are of interest. The engine deterioration is also considered in the aircraft performance monitoring systems that are provided by the aircraft manufacturers (Airbus, 2008, p. 18). Compared to them the present model has the advantage of being more flexible in studying various cases of aircraft operations without the need of real flight experiments.

To demonstrate the applicability of this approach in the development of airline fuel conservation strategy, a simplified scenario is investigated, and the fuel saving potential is evaluated.

1. Model description

1.1. Aircraft and engine base models

The aircraft and engine models were initially developed as a part of a PhD project, aimed at evaluation of the fuel saving potential of alternative types of auxiliary power units in commercial transport aircraft (Serbezov, 2012). The aircraft performance model is similar to the Eurocontrol BADA, but unlike BADA it does not need to replicate a large number of aircraft types. This gave an opportunity to optimize the calculation algorithms for the particular aircraft type and to use more complicated engine model. Also the approach for the identification of aircraft and engine characteristics was different.

The Airbus A319 with CFM56-5B was chosen as a base aircraft for the study. The characteristics of the base aircraft and its engines were identified with the use of records of regular flights from the aircraft digital flight data recorder together with data from various open source technical publications. The aircraft characteristics were incorporated in a flight dynamics performance model, realized in Matlab.

The model uses the classical expression of the airplane drag polar:

$$C_{xa} = C_{xa0} + A \cdot C_{za}^{2}, \qquad (1)$$

where C_{xa} is the aerodynamic drag coefficient, C_{xa0} is the zero lift drag coefficient, C_{xa} is the lift coefficient and A is an empirically calculated coefficient that characterize the induced drag. Booth C_{xa0} and A are calculated for the wing in "clean", take-off and landing configurations of the high-lift devices.

The aircraft performance model is based on the numerical solution of the following system of ordinary differential equations (ODE):

$$m\frac{dV}{dt} = F_x - C_{xa}\frac{\rho V^2}{2}S - m \cdot g \cdot \sin\gamma, \qquad (2)$$

$$\frac{dh}{dt} = V \cdot \sin \gamma, \tag{3}$$

$$\frac{dL}{dt} = V \cdot \sqrt{1 - \sin^2 \gamma},\tag{4}$$

$$\frac{dm}{dt} = -sfc \cdot F_x, \qquad (5)$$

where V is the airspeed, t is the time, F_x is the engines thrust force, ρ is the density of the air, S is the wing area, m is the mass of the aircraft, γ is the trajectory angle, h is the altitude above the mean sea level, L is the distance and sfc is the specific fuel consumption of the engines.

To solve the ODE system, at every step of the numerical solution an algebraic system consisting of equations 1, 2 and the following equations is to be solved:

$$0 = C_{za} \frac{\rho V^2}{2} S - mg \sqrt{1 - \sin^2 \gamma}, \qquad (6)$$

$$\sin\gamma = \frac{dh}{dt} \cdot \frac{1}{V} = \frac{dh}{dV} \cdot \frac{dV}{dt} \cdot \frac{1}{V}.$$
(7)

In cruise flight $\sin \gamma = 0$ and V = const (usually corresponding to M = 0.78), F_x is calculated from equation 2 and sfc is obtained from the engine model with respect to h, V and F_x , which significantly simplifies the calculations. Equations 3 and 7 are not used.

At climb and descend it is assumed that the power levers of the engines are at fixed setting (usually maximum climb or idle) and the flight is with constant calibrated air speed (CAS) or Mach number (M) (see Figure 2). CAS, M, V and h are interconnected through the International Standard Atmosphere (ISA). This relation makes possible to calculate numerically the derivative dh/dV and then to find γ , C_{za} and dV/dt by solving the system of equations 1, 2, 6 and 7.

The turbofan engine model was developed together with the aircraft model, using the same information sources. The model is based on the relation between the relative corrected fan speed (n_{1cor}) of the engine and the corrected thrust with regard to the flight Mach number. This makes possible to include in the model the maximum and minimum fan speed limitations that are imposed by the engine control system with respect to the altitude, Mach number, wing high-lift devices and landing gear position. More indepth description of the engine model can be found in (Serbezov, 2010).

Using Gasturb11 gas turbine performance software, a supplementary model was developed to account for the influence of the external factors on the engine performance (Serbezov, 2011). This supplementary model allows easy modifications to include the simulation of the engine deterioration.

1.2. Engine performance deterioration simulation

The turbofan engine performance deterioration is a complex phenomenon. Its main causes and mechanisms are described in Meher-Homji et al. (2001) and Zaita et al. (1997). The current research does not have the objective to model the engine deterioration itself, rather to model its effects on the overall aircraft performance. The turbo machinery deterioration is expressed as efficiency loss of the fan, the low and high pressure compressors (LPC and HPC), the combustor, the high and low pressure turbines (HPT and LPT). This makes possible significant simplification of the simulation. The following assumptions are made:

- To limit the effects of changing the operating point of the engine in the base turbofan engine model, it is decided to simulate efficiency loss of only 1% for the HPC and HPT. This condition reflects the early stages of the engine deterioration. For simulation of the impact of higher levels of the engine deterioration on the aircraft performance, a more complicated engine model will be needed;
- 2. To cover the entire engine operating envelope in common flight operations, while keeping a relatively small number of simulations the following combinations of altitude and Mach number are simulated (Table 1):

No.	Stage of the flight	Altitude, m	Mach number
1.	Cruise, initial descend	10668 (FL350)	0.8
2.	Final climb, cruise	10668	0.7
3.	Climb, descend	6069 (FL 200)	0.55 (CAS 250 kts)
4.	Ground operations, take off	0	0

Table 1. Flight conditions of the simulation

3. For every flight condition stated in Table 1 a set of engine simulations is made to cover the possible operating modes of the engines from full power (take off) to idle (Table 2):

Flight condition	Relative HP spool speed						
No.	1	0.95	0.9	0.8	0.75	0.7	0.6
1.	Х	Х	Х	-	-	-	-
2.	Х	Х	Х	Х	Х	-	-
3.	Х	Х	Х	Х	-	Х	-
4.	Х	Х	Х	Х	-	Х	Х

Table 2. Simulation points

4. The control law of the engine has a significant effect on the way the fuel flow changes with the changes in the internal engine efficiencies. The control systems of modern turbofan engines use complex laws that are not publicly known. In case of flight in steady conditions and changes caused only by engine deterioration these laws can be substituted by three basic laws.

At maximum power (take off) and climb operating modes it is assumed that the engine is limited by the maximum allowed fan speed (n_1) for the given ambient conditions. The simulation of the deteriorated engine shall be performed with the same n_1 as that of the base engine. The thrust of the deteriorated engine will vary, which shall be considered in the airplane flight model;

At cruise operating modes it is assumed that the engine provides the thrust required to keep the airplane steady flight. The simulation of the deteriorated engine shall be performed with the same thrust as that of the base engine;

At idle operation the HPC speed (n_2) is kept above its lower limit in order to allow stable engine running. The simulation shall be performed with the same n_2 for the base and the deteriorated engines.

The simulations of all of the cases with and without deterioration were performed in GasTurb 11 gas turbine performance simulation software, using the supplementary engine model described in (Serbezov, 2011). Further processing of the results was performed in Excel. The main parameters of interest were:

The relative increase of the specific fuel consumption of the deteriorated engine (d_sfc) in percents for all of the simulation points (equation 8):

$$d_sfc = \frac{sfc_{\det er} - sfc_{base}}{sfc_{base}} \cdot 100,\%, \qquad (8)$$

where sfc is the specific fuel consumption, subscripts 'deter' and 'base' apply for the deteriorated and the base engine respectively;

The relative change of the fan speed (d_n_1) , calculated in a way similar to d_sfc, to analyse the change of the engine operating point (equation 9):

$$d_{-}n_{1} = \frac{n_{1\text{det}er} - n_{1\text{base}}}{n_{1\text{base}}} \cdot 100,\%.$$
(9)

Relative corrected fan speed (n_{1cor}) as a determinant of the operating point, used in the base engine model.

The results for the d_sfc and d_ n_1 are shown in Table 3 and Table 4 respectively. Figure 1 shows the d_sfc as a function of the n_{1cor} for all of the calculation cases.

Table 3. Relative increase of the specific fuel consumption of the deteriorated engine d_sfc, %

Flt. cond.		đ					
No.	1	0.95	0.9	0.8	0.75	0.7	0.6
1.	0.35	0.97	1.86				
2.	0.23	0.84	1.75	2.48	2.42		
3.	0.79	1.65	2.22	2.90		2.58	
4.	1.68	2.27	2.7	2.90		3.06	3.40

Table 4. Relative change of the fan speed d_n_1 (absolute value), %

Flt. cond.	Relative HP spool speed							
No.	1	0.95	0.9	0.8	0.75	0.7	0.6	
1.	0.14	0.18	0.18					
2.	0.16	0.16	0.17	0.20	0.25			
3.	0.14	0.13	0.12	0.28		0.20		
4.	0.04	0.01	0.00	0.02		0.02	0.03	



Figure 1. The relative increase of the specific fuel consumption of the deteriorated engine (d_sfc) as a function of the relative change of the fan speed (d_ n_1)

Based on the simulation results the following conclusions are made:

- The relative change of the fan speed is small enough to allow a simple integration of the deterioration data in the general turbofan engine model by correcting the fuel consumption without affecting the engine thrust;
- The relative increase of the specific fuel consumption at the level of deterioration selected (Table 3) is sensible enough to impact the aircraft performance model, and is suitable for further simulations for the purposes of fuel conservation strategies;
- There is a clear correlation between the d_sfc and n_{1cor} irrespective of the Mach number, which further simplifies the task of integration of the deterioration data in the general turbofan engine model.

1.3. Integration of Engine Deterioration Data in the Base Model of the Engine

Based on the conclusions of the previous paragraph, it was assumed that the modification of the engine model to account for the given level of deterioration can be accomplished just by correcting the fuel consumption results calculated by the algorithm described in (Serbezov, 2011). To do this, the d_sfc – n_{1cor} correlation is interpolated with a polynomial of 4 degree, using least square method in Matlab. The resulting function 'd_sfc fit' is shown with brown dashed line in Figure 1. The fit produces a R square value of 0.976 which is assumed satisfactory for the purpose. It can be also observed on Figure 1 that the dispersion of the data is mainly in the region of engines idle operation which has a small impact on the overall engine fuel consumption during standard flights. The deterioration correction of the calculated fuel consumption is accomplished by applying the following equation at the end of the calculation algorithm:

$$w_f = w_f \cdot \left(1 + \frac{d_{-} sfc(n_{1cor})}{100}\right),\tag{10}$$

where w_f is the fuel flow to the engine in kg/s and $d_sfc(n_{1cor})$ is the polynomial fit function used to calculate d_sfc for the current engine operating mode.

2. Case studies – engine deterioration impact on optimal flight level

The case study was performed with the intent to identify possible fuel savings that can be achieved if the engine deterioration is taken in to account in the flight planning process and the optimum cruise flight level is determined with respect to it. The optimum cruise flight level is selected during the flight planning, based on the flight distance, the environmental conditions and the operating weight of the aircraft. It is known that for instrument flight rules (IFR) flights with Reduced Vertical Separation Minima and Semicircular rule the flight level in feet should be odd thousands for eastbound flights (e.g. FL350, FL370 ...) or even thousands for westbound flights (... FL340, FL360 ...).

To achieve this several simulations were made for flights at distances of 2000, 2500 and 3000 km, at flight levels FL390 (the maximal allowed FL for the most of the A319 modifications) and FL370, with different payload.

2.1. Flight scenario general characteristics

All of the flight simulations were made with a flight profile that matches the standard operating procedures and practices as described in Airbus A319 Flight crew operations manual (FCOM). The flight profile with some of its parameters is presented on Figure 2. On Figure 2 CAS is the calibrated air speed in knots, and the altitude is given as Flight level (FL) in thousands of feet. The departure and destination airports are on the sea level at standard meteorological conditions. Take off speed V2 and final approach speed Vapp are calculated in accordance with the A319 FCOM, based on the airport altitude, ambient temperature and the aircraft weight.

The main input variables for every simulation are:

- the deterioration status of the engines 'deteriorated' or 'new';
- the load factor in percent of the maximum payload (18000 kg);
- the cruise flight level altitude in ft;
- the flight distance in km.

During the simulation first the reserve fuel is calculated based on the deterioration status and the load factor. Secondly the trip fuel is calculated based on the aircraft mass including the reserve fuel, the cruise flight level altitude and the flight distance. After the simulations in Matlab the results are further processed in Excel.



Figure 2. Elements of the simulated flight profile

2.2. Simulation results

The trip fuel simulation results are presented on Tables 5–10. The lower trip fuel mass, corresponding to the optimum flight level is given in bold.

	Table 5.	Trip	fuel	in	kg,	2000	km	flight,	'New'
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Elight Laval	Load factor, %				
Flight Level	100	80	70		
390	6432.1	6060.2	5884.7		
370	6406.3	6076.7	5922.1		

Table 6. Trip fuel in kg, 2000 km flight, 'Deteriorated'

Flight Level	Load factor, %				
riight Level	100	80	70		
390	6556.6	6167.0	5980.9		
370	6510.5	6161.2	5996.1		

Table 7. Trip fuel in kg, 2500 km flight, 'New'

Flight Level	Load factor, %				
	90	85	80		
390	7692.4	7578.2	7464.4		
370	7681.0	7580.9	7480.3		

Table 8. Trip fuel in kg, 2500 km flight, 'Deteriorated'

Elizht Laval	Load factor, %				
Flight Level	90	85	80		
390	7825.2	7706.7	7588.6		
370	7797.7	7693.1	7677.3		

Table 9. Trip fuel in kg, 3000 km flight, 'New
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Flight Level	Load factor, %				
Flight Level	85	80	75		
390	9034.5	8896.7	8757.414		
370	9026.9	8910.6	8790.891		

Table 10. Trip fuel in kg, 3000 km flight, 'Deteriorated'

Flight Level	Load factor, %				
Tlight Level	85	80	75		
390	9178.3	9036.2	8894.9		
370	9161.7	9036.1	8912.9		

2.3. Results interpretation

For further analysis the difference between the mass of FL370 and FL390 trip fuel ($\Delta m_{TRIP \ FUEL}$) was calculated. Positive $\Delta m_{TRIP \ FUEL}$ indicates that FL370 is the optimal FL, and negative $\Delta m_{TRIP \ FUEL}$ – FL370 is the optimal. The results for 3000 km flight are presented on Figure 3.



Figure 3. Fuel saving potential for 'Deteriorated engines' airplane at 3000 km flight distance

Figure 3 shows that $\Delta m_{TRIP \ FUEL}$ continuously decreases with the increase of the load factor and the relation can be assumed as linear. This fact allows the linear interpolation of $\Delta m_{TRIP \ FUEL}$ and the determination of the load factor at which $\Delta m_{TRIP \ FUEL}$ becomes zero, so the optimum flight level changes. The results of the calculations for 3000 km flight are shown also on Figure 3.

The interval between the 'deteriorated' and 'new' airplanes load factor of optimum FL switch ('Zone of potential fuel saving' on Figure 3) is the interval where potential fuel conservation can be achieved if the flight planning is performed with respect to the engines performance deterioration. The maximal fuel conservation will be just before the 'new' airplane optimum FL switches from the higher to the lower FL, and it will be equal to the absolute value of $\Delta m_{TRIP FUEL}$ 'deteriorated' at that point. The summarized results for all of the simulated flight distances are shown in Table 11:

Table 11. Trip fuel in kg, 2000 km flight, 'New'

Flight Distance, km	Load factor of optimum FL switching, %		Maximum fuel saving,	Relative fuel
	'deteriorated'	'new'	kg	saving, %
2000	77.3	88.0	22.4	0.36
2500	80	86	16.4	0.21
3000	80	83.2	10.8	0.12

Table 11 reviles a clear trend for the 'Zone of potential fuel saving' and the maximum fuel saving to increase with the decrease of the flight distance. This behaviour can be explained with the opposite effects that have the climb and cruise on the total amount of the trip fuel. The climb to higher altitude increases the fuel burn, but cruise flight at this altitude reduces it. At shorter distances the relative effect of the climb leg will increase, which will further increase the effect of the engine deterioration.

Given the targets of 2–6% fuel conservation for the present airlines fuel conservation strategies, the potential fuel saving of 0.1–0.4% per flight at relatively low levels of engine deterioration is a sensible value. It is also important that the load factor interval of the potential fuel saving

intersects with the average passenger load factor of commercial airlines worldwide from 2005 to 2019 (75.2–82.1%) (Statista, 2019).

Conclusions

The present work shows the applicability of the simulation approach to assess the engines performance deterioration as a factor in the airlines operational fuel conservation. The estimate of the fuel saving potential is based on comparison of the results of the same model with only engine fuel consumption changed and all other conditions equal. For this reason it can be concluded that the estimate depends insignificantly from the magnitude of the simplifying assumptions of the model and it represents correctly the 'real' influence of the engine deterioration.

Although only the factor of engine deterioration was addressed, a great variety of other operational factors can be introduced in the model easily. The performed simulations reviled the following:

- If the flight planning for the 'deteriorated' airplane is done with performance data for the 'new' one, for a given flight distance there will be a certain interval of load factors in which a not optimal FL will be chosen and this will lead to excess fuel burn.
- The assessed potential fuel saving of 0.1–0.4% per flight at relatively low levels of engine deterioration is a sensible value.
- The load factor interval of the potential fuel saving intersects with the average passenger load factor of commercial airlines.

The demonstrated approach can be assessed as a promising one, but to get a practical application it should be developed towards the creation of 'personal' numerical models of every single aircraft and its power plant, combined with reliable methods of keeping it adequate to the actual performance deterioration, using the available data from the aircraft flight data recorders and other sources. Based on this conclusion a further research in the area is highly recommended.

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